Chapter 7

Control Charts for Attributes

Learning Objectives

After completing this chapter you should be able to:

1. Understand the statistical basis of attributes control charts
2. Know how to design attributes control charts
3. Know how to set up and use the *p* chart for fraction nonconforming
4. Know how to set up and use the *np* control chart for the number of nonconforming items
5. Know how to set up and use the *c* control chart for defects
6. Know how to set up and use the *u* control chart for defects per unit
7. Use attributes control charts with variable sample size
8. Understand the advantages and disadvantages of attributes versus variables control charts
9. Understand the rational subgroup concept for attributes control charts
10. Determine the average run length for attributes control charts

Important Terms and Concepts

Attribute data
Average run length for attributes control charts
Cause-and-effect diagram
Choice between attributes and variables data
Control chart for defects or nonconformities per unit or *u* chart
Control chart for fraction nonconforming or *p* chart
Control chart for nonconformities or *c* chart
Control chart for number nonconforming or *np* chart
Defect
Defective
Demerit systems for attribute data
Design of attributes control charts
Fraction defective
Fraction nonconforming
Nonconformity
Operating characteristic curve for the *c* and *u* charts
Operating characteristic curve for the *p* chart
Pareto chart
Standardized control charts
Time between occurrence control charts
Variable sample size for attributes control chart

EXERCISES

New exercises are denoted by "☺".

Minitab® Notes:

1. The Minitab convention for determining whether a point is out of control is: (1) if a plot point is within the control limits, it is in control, or (2) if a plot point is on or beyond the limits, it is out of control.
2. Minitab defines some sensitizing rules for control charts differently than the standard rules. In particular, a run of *n* consecutive points on one side of the center line is defined as 9 points, not 8. This can be changed in dialog boxes, or under Tools > Options > Control Charts and Quality Tools > Tests.

7.1. ☺

A financial services company monitors loan applications. Every day 50 applications are assessed for the accuracy of the information on the form. Results for 20 days are $\sum_{i=1}^{20} D_i = 46$ where D_i is the number of loans on the *i*th day that re determined to have at least one error. What are the center line and control limits on the fraction nonconforming control chart?

$$n = 50;\ \ m = 20;\ \ \sum_{i=1}^{20} D_i = 46;\ \ \bar{p} = \frac{\sum_{i=1}^{m} D_i}{mn} = \frac{46}{20(50)} = 0.046$$

$$\text{UCL}_p = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.046 + 3\sqrt{\frac{0.046(1-0.046)}{50}} = 0.046 + 0.089 = 0.135$$

$$\text{LCL}_p = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.046 - 3\sqrt{\frac{0.046(1-0.046)}{50}} = 0.046 - 0.089 \Rightarrow 0$$

7.2. ☺

Do points that plot below the lower control limit on a fraction nonconforming control chart (assuming that the LCL > 0) always mean that there has been an improvement in process quality? Discuss your answer in the context of a specific situation.

No, points plotting below the lower control limit do not always indicate an improvement in process quality. As with any sample exceeding control limits, they may be caused by inspection process errors related to calibration, use of equipment, poor training, or inexperience. Points below the lower control limit may also result from deliberate actions to pass nonconforming material or record fictitious data – this is a more subtle consequence of categorizing product as conforming or nonconforming, versus identifying a characteristic for numerical measurement.

7.3. ☺

Tale 7E.1 contains data on examination of medical insurance claims. Every day 50 claims were examined.

TABLE 7E.1
Medical Insurance Claim Data for Exercise 7.3

Day	Number Nonconforming	Day	Number Nonconforming
1	0	11	6
2	3	12	4
3	4	13	8
4	6	14	0
5	5	15	7
6	2	16	20
7	8	17	6
8	9	18	1
9	4	19	5
10	2	20	7

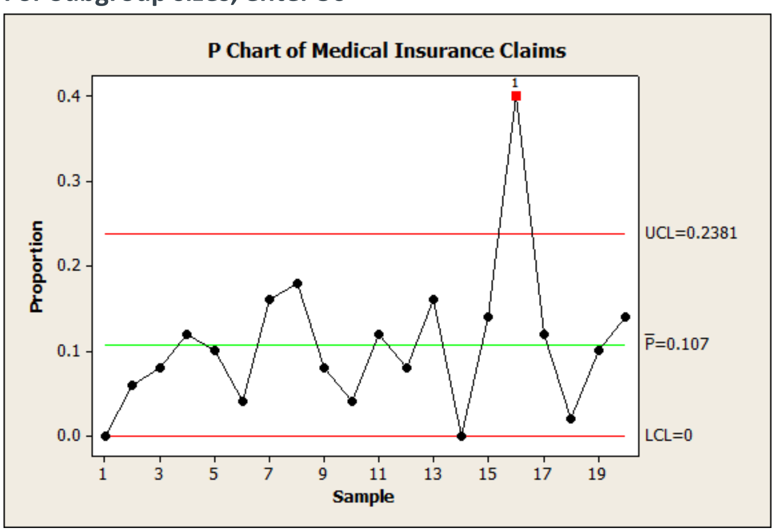
(a) Set up the fraction nonconforming control chart for this process. Plot the preliminary data in Table 7E.1 on the chart. Is the process in statistical control?

$$n=50;\quad m=20;\quad \sum_{i=1}^{20} D_i = 107;\quad \bar{p} = \frac{\sum_{i=1}^{m} D_i}{mn} = \frac{107}{20(50)} = 0.107$$

$$\text{UCL}_p = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.107 + 3\sqrt{\frac{0.107(1-0.107)}{50}} = 0.107 + 0.131 = 0.238$$

$$\text{LCL}_p = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.107 - 3\sqrt{\frac{0.107(1-0.107)}{50}} = 0.107 - 0.131 \Rightarrow 0$$

MTB > Stat > Control Charts > Attributes Charts > P
For Subgroup sizes, enter 50



Test Results for P Chart of Ex7-3Num

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  16
```

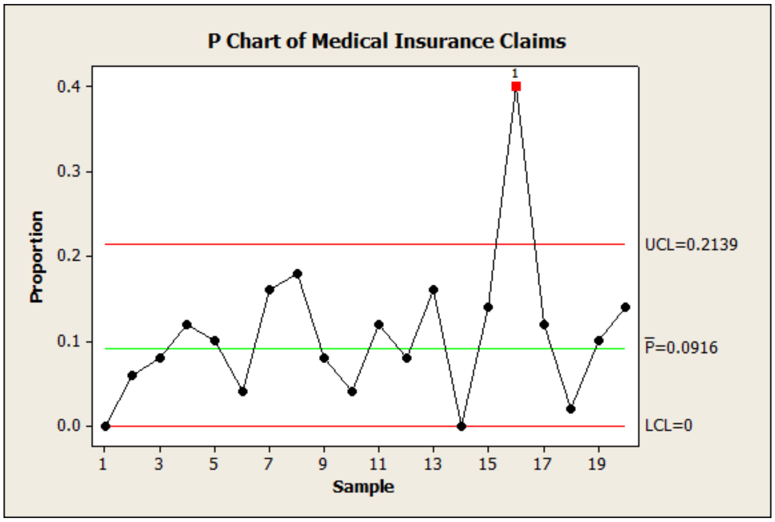
The process is not in statistical control. Day 16 exceeds the upper control limit, signaling a potential assignable cause.

7.3. continued

(b) Assume that assignable causes can be found for any out-of-control points on this chart. What center line and control limits should be used for process monitoring in the next period?

MTB > Stat > Control Charts > Attributes Charts > P

Under P Chart Options, click on the Estimate tab, and Omit subgroup 16 when estimating parameters.



Test Results for P Chart of Ex7-3Num

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  16
```

No additional days signal out of control. Use UCL = 0.2139, CL = 0.0916 and LCL = 0 to monitor the process.

7.4. ☺

The fraction nonconforming control chart in Exercise 7.3 has an LCL of zero. Assume that the revised control chart in part (b) of that exercise has a reliable estimate of the process fraction nonconforming. What sample size should be used if you want to ensure that the LCL > 0?

To choose *n* large enough so that the lower control limit is positive and assuming three-sigma control limits are to be used:

$$n > \frac{(1-p)}{p}L^2 = \frac{(1-0.1053)}{0.1053}3^2 = 76.47 \approx 75$$

7.5. ☺

The commercial loan operation of a financial institution has a standard for processing new loan applications in 24 hours. Table 7E.2 shows the number of applications processed each day for the last 20 days and the number of applications that required more than 24 hours to complete.

TABLE 7E.2

Loan Application Data for Exercise 7.5

Day	Number of Applications	Number Late	Day	Number of Applications	Number Late
1	200	3	11	219	0
2	250	4	12	238	10
3	240	2	13	250	4
4	300	5	14	302	6
5	200	2	15	219	20
6	250	4	16	246	3
7	246	3	17	251	6
8	258	5	18	273	7
9	275	2	19	245	3
10	274	1	20	260	1

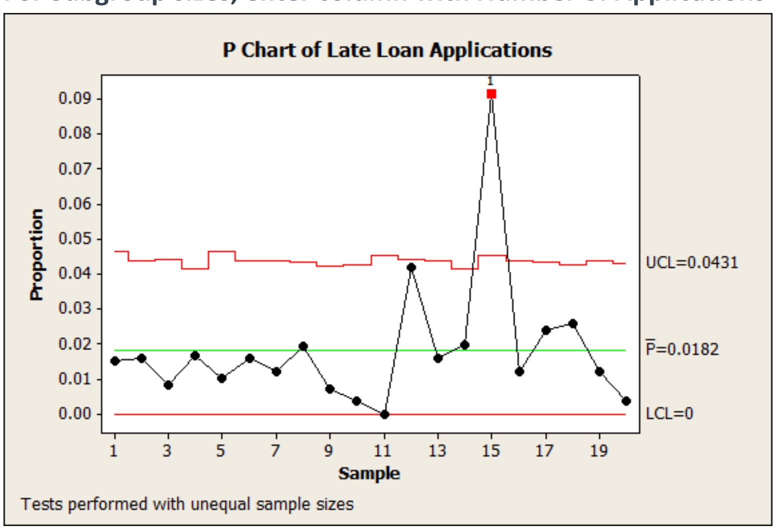
(a) Set up the fraction nonconforming control chart for this process. Use the variable-width control limit approach. Plot the preliminary data in table 7E.2 on the chart. Is the process in statistical control?

$$\bar{p} = \frac{\sum_{i=1}^{20} D_i}{\sum_{i=1}^{20} n_i} = \frac{91}{4996} = 0.0182$$

$$\text{Control Limits}_i = \bar{p} \pm 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}} = 0.0182 \pm 3\sqrt{\frac{0.0182(1-0.0182)}{n_i}} = 0.0182 \pm 3\sqrt{\frac{0.0179}{n_i}} = 0.0182 \pm \frac{0.401}{\sqrt{n_i}}$$

MTB > Stat > Control Charts > Attributes Charts > P

For Subgroup sizes, enter column with Number of Applications



Test Results for P Chart of Ex7-5Late

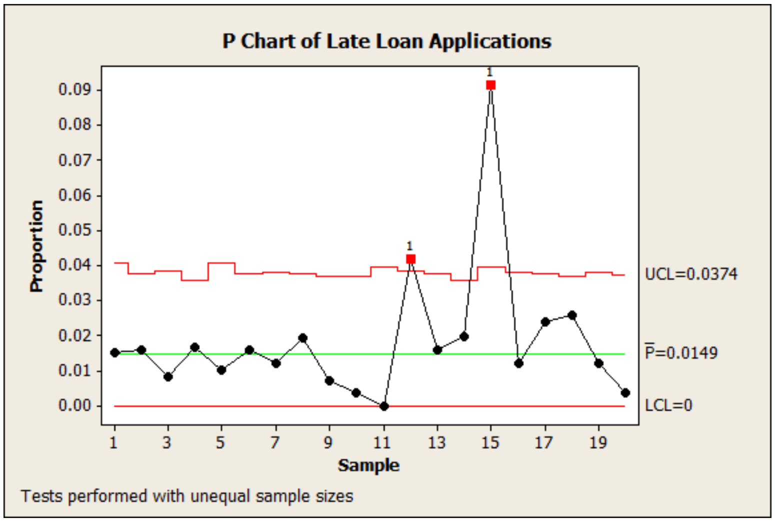
```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  15
```

The process is not in statistical control, with Day 15 exceeding the upper control limit.

7.5. continued

(b) Assume that assignable causes can be found for any out-of-control points on this chart. What center line should be used for process monitoring in the next period, and how should the control limits be calculated?

MTB > Stat > Control Charts > Attributes Charts > P
Under P Chart Options, click on the Estimate tab, and Omit subgroup 15 when estimating parameters.



Test Results for P Chart of Ex7-5Late

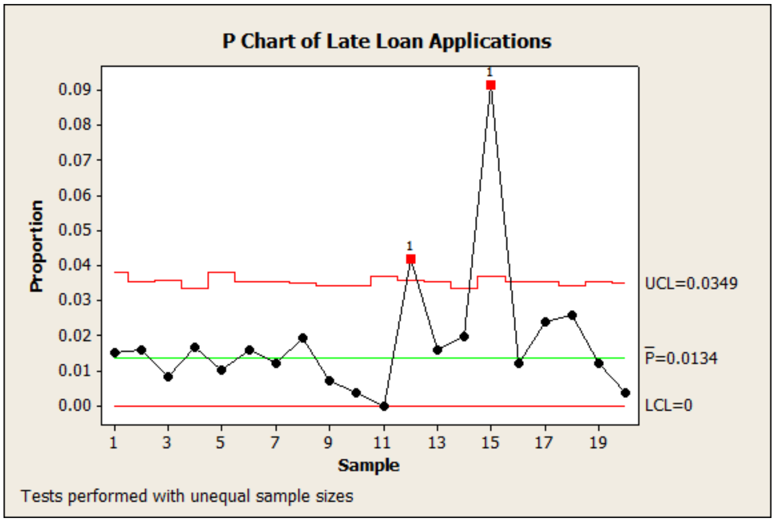
```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  12, 15
```

Note that Day 12 now signals out of control. Assuming that an assignable cause can be found, also eliminate this day when calculating control limits.

7.5. continued

MTB > Stat > Control Charts > Attributes Charts > P

Under P Chart Options, click on the Estimate tab, and Omit subgroups 12 and 15 when estimating parameters.



Test Results for P Chart of Ex7-5Late

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  12, 15
```

No additional days signal out of control. Use UCL = 0.0349, CL = 0.0134 and LCL = 0 to monitor the process.

7.6. ☺

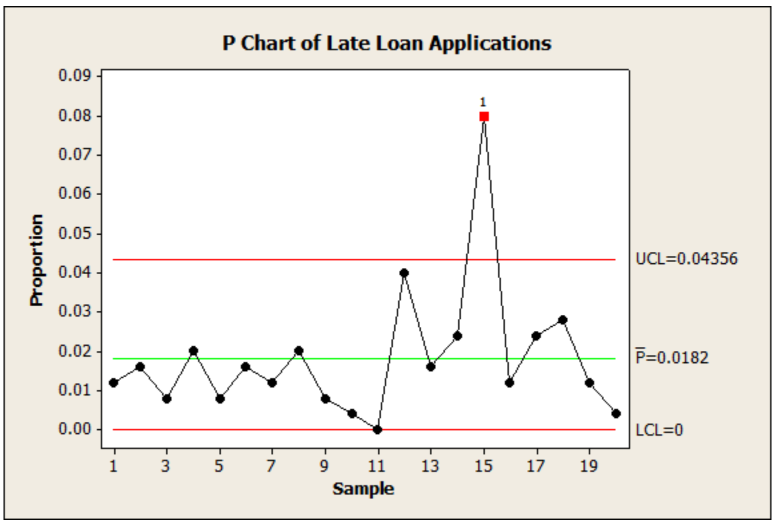
Reconsider the loan application data in Table 7E.2. Set up the fraction nonconforming control chart for this process. Use the average sample size control limit approach. Plot the preliminary data in Table 7E.2 on the chart. Is the process in statistical control? Compare this control chart to the one based on variable-width control limits in Exercise 7.5.

$$\bar{p} = \frac{\sum_{i=1}^{20} D_i}{\sum_{i=1}^{20} n_i} = \frac{91}{4996} = 0.0182; \quad \bar{n} = \frac{\sum_{i=1}^{20} n_i}{20} = \frac{4996}{20} = 249.8 \approx 250$$

$$\text{Control Limits}_i = \bar{p} \pm 3\sqrt{\frac{\bar{p}(1-\bar{p})}{\bar{n}}} = 0.0182 \pm 3\sqrt{\frac{0.0182(1-0.0182)}{250}} = 0.0182 \pm 0.0254 = [0.0436, 0]$$

MTB > Stat > Control Charts > Attributes Charts > P

For Subgroup sizes, enter the average sample size of 250



Test Results for P Chart of Ex7-5Late

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  15
```

Similar to the control chart with variable-width limits, Day 15 signals out of control, with the remaining days in statistical control

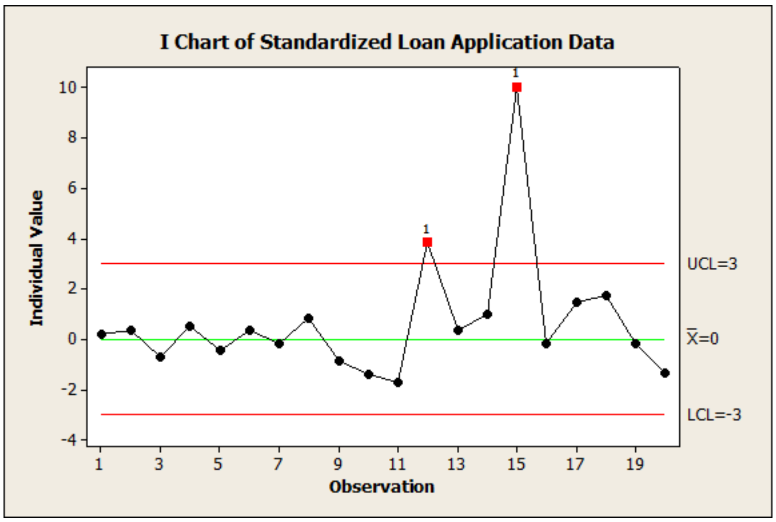
7.7. ☺

Reconsider the loan application data in Table 7E.2. Set up the fraction nonconforming control chart for this process. Use the standardized control chart approach. Plot the preliminary data in Table 7E.2 on the chart. Is the process in statistical control? Compare this control chart to the one based on variable-width control limits in Exercise 7.5.

Use the process fraction nonconforming in the in-control state, $\bar{p} = 0.0134$, and calculate $Z_i = \dfrac{\hat{p}_i - \bar{p}}{\sqrt{\dfrac{\bar{p}(1-\bar{p})}{n_i}}}$

MTB > Stat > Control Charts > Variables Charts for Individuals > Individuals

Under I Chart Options, on the Parameters tab, enter Mean = 0 and Standard deviation = 1



Test Results for I Chart of Ex7-7Zi

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  12, 15
```

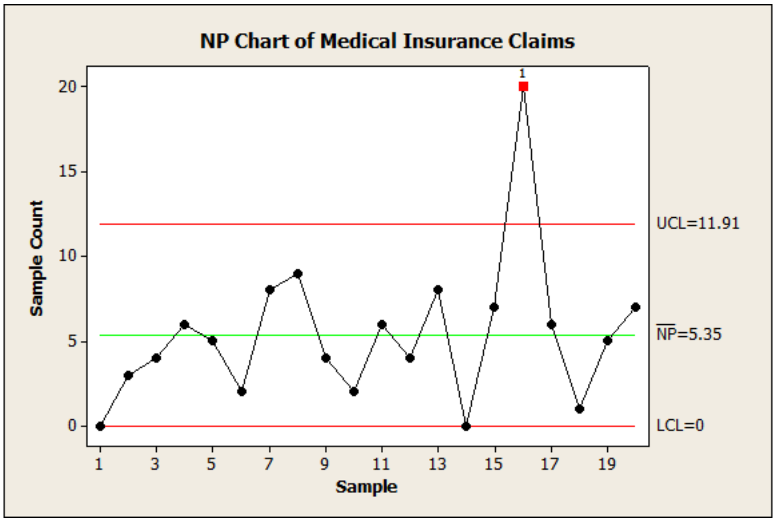
Similar to the final control chart with variable-width limits, both Days 12 and 15 signal out of control, with the remaining days in statistical control.

7.8. ☺

Reconsider the insurance claim data in Table 7E.1. Set up an *np* control chart for this data and plot the data from Table 7E.1 on this chart. Compare this to the fraction nonconforming control chart in Exercise 7.3.

MTB > Stat > Control Charts > Attributes Charts > NP

For Subgroup sizes, enter 50



Test Results for NP Chart of Ex7-3Num

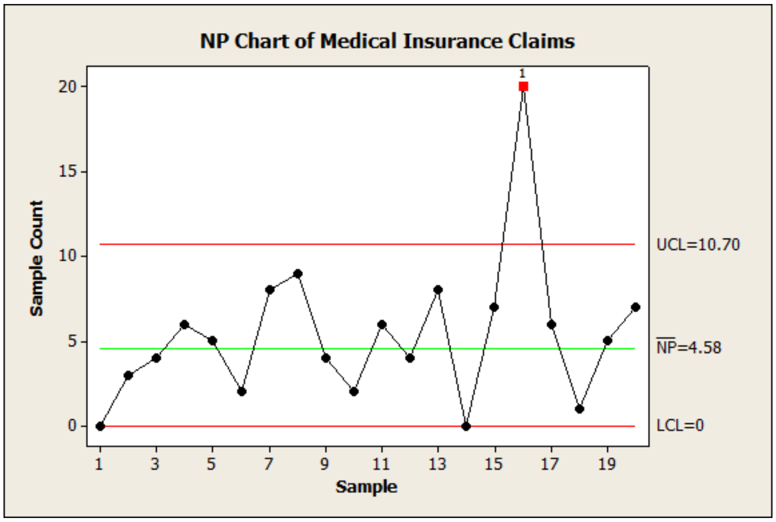
```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  16
```

Similar to the *p* chart, the process is not in statistical control. Day 16 exceeds the upper control limit, signaling a potential assignable cause. Removing Day 16 from the control limit calculation gives $\overline{np} = 4.58$ and an UCL = 10.7, which can be used to monitor the process going forward.

MTB > Stat > Control Charts > Attributes Charts > NP

For Subgroup sizes, enter 50

To exclude Day 16, select NP Chart Options, Estimate tab, and Omit subgroup 16



7.9.

The data in Table 7E.3 give the number of nonconforming bearing and seal assemblies in samples of size 100. Construct a fraction nonconforming control chart for these data. If any points plot out of control, assume that assignable causes can be found and determine the revised control limits.

TABLE 7E.3
Data for Exercise 7.9

Sample Number	Number of Nonconforming Assemblies	Sample Number	Number of Nonconforming Assemblies
1	7	11	6
2	4	12	15
3	1	13	0
4	3	14	9
5	6	15	5
6	8	16	1
7	10	17	4
8	5	18	5
9	2	19	7
10	7	20	12

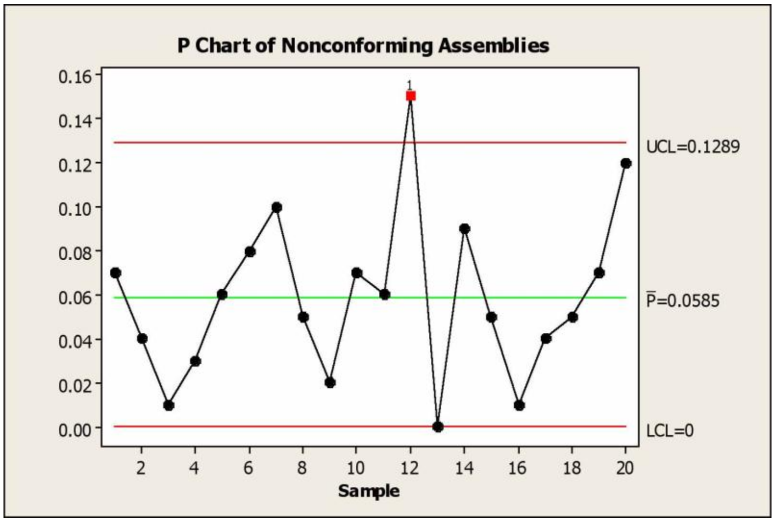
$$n = 100;\quad m = 20;\quad \sum_{i=1}^{m} D_i = 117;\quad \bar{p} = \frac{\sum_{i=1}^{m} D_i}{mn} = \frac{117}{20(100)} = 0.0585$$

$$\text{UCL}_p = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.0585 + 3\sqrt{\frac{0.0585(1-0.0585)}{100}} = 0.1289$$

$$\text{LCL}_p = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.0585 - 3\sqrt{\frac{0.0585(1-0.0585)}{100}} = 0.0585 - 0.0704 \Rightarrow 0$$

MTB > Stat > Control Charts > Attributes Charts > P

For Subgroup sizes, enter 100



Test Results for P Chart of Ex7.1Num

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  12
```

7.9. continued

Sample 12 is out-of-control, so remove from control limit calculation:

$$n = 100;\ \ m = 19;\ \ \sum_{i=1}^{m} D_i = 102;\ \ \overline{p} = \frac{\sum_{i=1}^{m} D_i}{mn} = \frac{102}{19(100)} = 0.0537$$

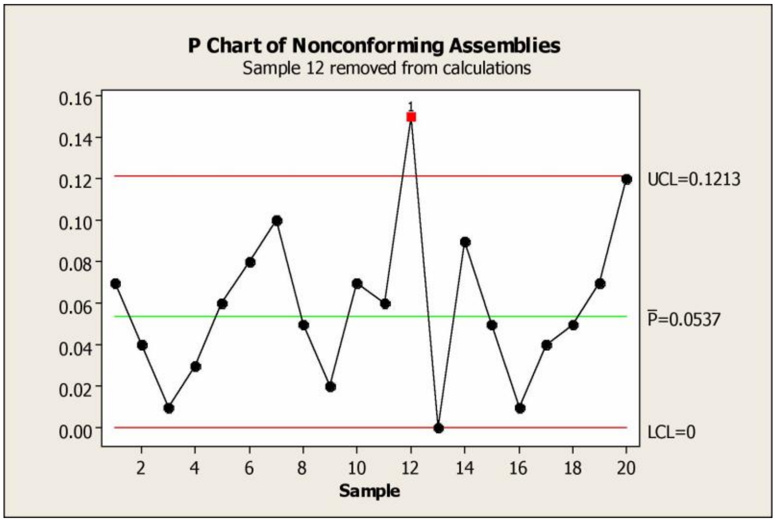
$$\text{UCL}_p = 0.0537 + 3\sqrt{\frac{0.0537(1-0.0537)}{100}} = 0.1213$$

$$\text{LCL}_p = 0.0537 - 3\sqrt{\frac{0.0537(1-0.0537)}{100}} = 0.0537 - 0.0676 \Rightarrow 0$$

MTB > Stat > Control Charts > Attributes Charts > P

For Subgroup sizes, enter 100

To exclude Sample 12, select P Chart Options, Estimate tab, and Omit subgroup 16



Test Results for P Chart of Ex7.1Num

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  12
```

No additional samples signal out of control. Use UCL = 0.1213, CL = 0.0537 and LCL = 0 to monitor the process.

7.10.

The number of nonconforming switches in samples of size 150 are shown in Table 7E.4. Construct a fraction nonconforming control chart for these data. Does the process appear to be in control? If not, assume that assignable causes can be found for all points outside the control limits and calculate the revised control limits.

TABLE 7E.4

Number of Nonconforming Switches for Exercise 7.10

Sample Number	Number of Nonconforming Switches	Sample Number	Number of Nonconforming Switches
1	8	11	6
2	1	12	0
3	3	13	4
4	0	14	0
5	2	15	3
6	4	16	1
7	0	17	15
8	1	18	2
9	10	19	3
10	6	20	0

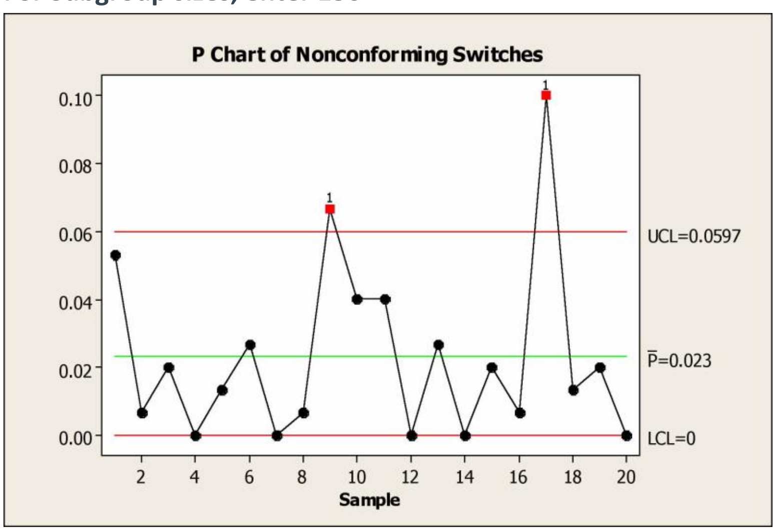
$$n = 150;\ \ m = 20;\ \ \sum_{i=1}^{m} D_i = 69;\ \ \bar{p} = \frac{\sum_{i=1}^{m} D_i}{mn} = \frac{69}{20(150)} = 0.0230$$

$$\text{UCL}_p = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.0230 + 3\sqrt{\frac{0.0230(1-0.0230)}{150}} = 0.0597$$

$$\text{LCL}_p = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.0230 - 3\sqrt{\frac{0.0230(1-0.0230)}{150}} = 0.0230 - 0.0367 \Rightarrow 0$$

MTB > Stat > Control Charts > Attributes Charts > P

For Subgroup sizes, enter 150



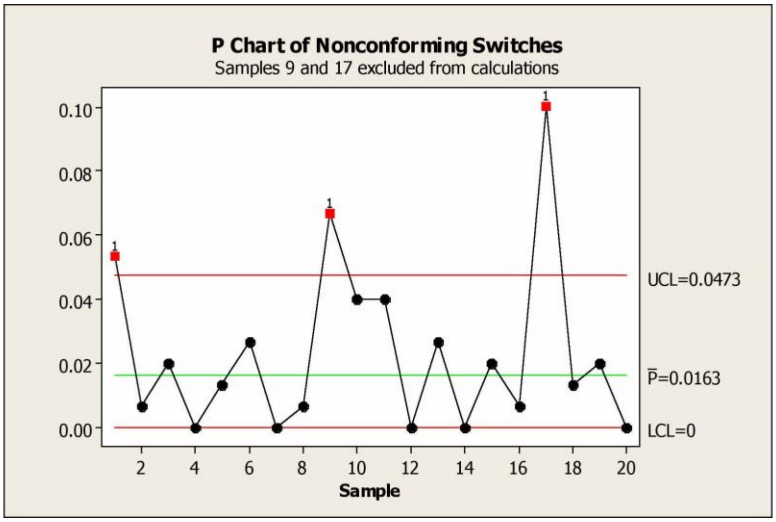
Test Results for P Chart of Ex7.2Num

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  9, 17
```

Samples 9 and 17 are out of control, so re-calculate control limits without samples 9 and 17:

7.10. continued

MTB > Stat > Control Charts > Attributes Charts > P
For Subgroup sizes, enter 150
To exclude Sample 12, select P Chart Options, Estimate tab, and Omit subgroups 9 and 17



Test Results for P Chart of Ex7.2Num

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  1, 9, 17
```

Also remove sample 1 from control limits calculation:

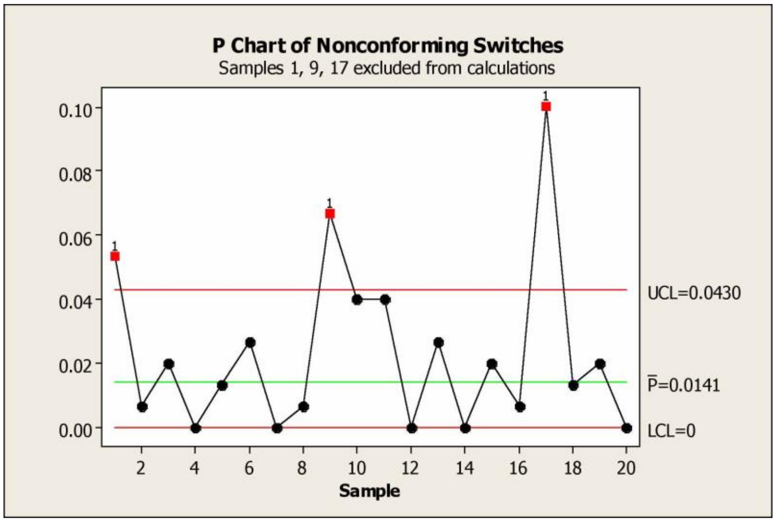
$$n = 150;\ \ m = 17;\ \ \sum_{i=1}^{m} D_i = 36;\ \ \bar{p} = \frac{\sum_{i=1}^{m} D_i}{mn} = \frac{36}{17(150)} = 0.0141$$

$$\text{UCL}_p = 0.0141 + 3\sqrt{\frac{0.0141(1-0.0141)}{150}} = 0.0430$$

$$\text{LCL}_p = 0.0141 - 3\sqrt{\frac{0.0141(1-0.0141)}{150}} = 0.0141 - 0.0289 \Rightarrow 0$$

7.10. continued

MTB > Stat > Control Charts > Attributes Charts > P
For Subgroup sizes, enter 150
To exclude Sample 12, select P Chart Options, Estimate tab, and Omit subgroups 1, 9, 17



Test Results for P Chart of Ex7.2Num

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  1, 9, 17
```

No additional samples signal out of control. Use UCL = 0.0430, CL = 0.0141 and LCL = 0 to monitor the process.

7.11.

The data in Table 7E.5 represent the results of inspecting all units of a personal computer produced for the last 10 days. Does the process appear to be in control?

TABLE 7E.5

Personal Computer Inspecting Results for Exercise 7.11

Day	Units Inspected	Nonconforming Units	Fraction Nonconforming
1	80	4	0.050
2	110	7	0.064
3	90	5	0.056
4	75	8	0.107
5	130	6	0.046
6	120	6	0.050
7	70	4	0.057
8	125	5	0.040
9	105	8	0.076
10	95	7	0.074

$$m = 10;\ \sum_{i=1}^{m} n_i = 1000;\ \sum_{i=1}^{m} D_i = 60;\ \bar{p} = \sum_{i=1}^{m} D_i \Big/ \sum_{i=1}^{m} n_i = 60/1000 = 0.06$$

$$\text{UCL}_i = \bar{p} + 3\sqrt{\bar{p}(1-\bar{p})/n_i} \text{ and } \text{LCL}_i = \max\{0, \bar{p} - 3\sqrt{\bar{p}(1-\bar{p})/n_i}\}$$

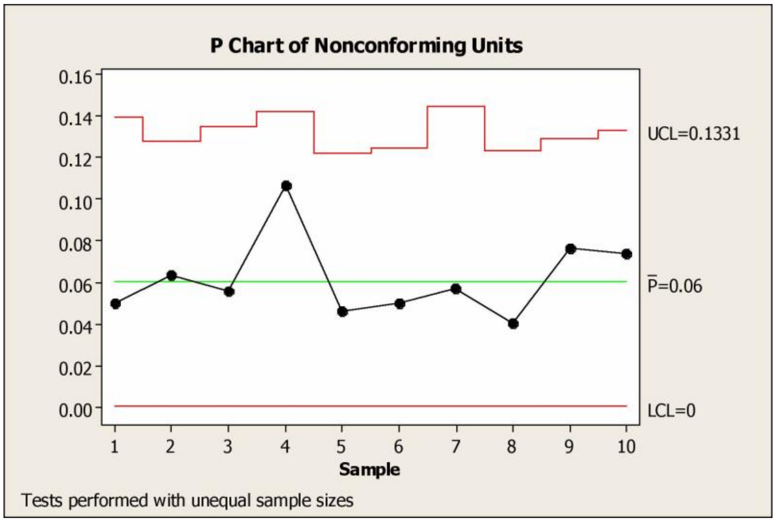
As an example, for n = 80:

$$\text{UCL}_1 = \bar{p} + 3\sqrt{\bar{p}(1-\bar{p})/n_1} = 0.06 + 3\sqrt{0.06(1-0.06)/80} = 0.1397$$

$$\text{LCL}_1 = \bar{p} - 3\sqrt{\bar{p}(1-\bar{p})/n_1} = 0.06 - 3\sqrt{0.06(1-0.06)/80} = 0.06 - 0.0797 \Rightarrow 0$$

MTB > Stat > Control Charts > Attributes Charts > P

For Subgroup sizes, enter column with Units Inspected



The process appears to be in statistical control.

7.12.

A payment process that reimburses members for out-of-network health expenses is to be controlled through use of a fraction nonconforming chart. Initially one sample of size 200 is taken each day for 20 days, and the results shown in Table 7E.6 are observed.

TABLE 7E.6
Nonconforming Unit Data for Exercise 7.12

Day	Nonconforming Units	Day	Nonconforming Units
1	3	11	2
2	2	12	4
3	4	13	1
4	2	14	3
5	5	15	6
6	2	16	0
7	1	17	1
8	2	18	2
9	0	19	3
10	5	20	2

(a) Establish a control chart to monitor future performance.

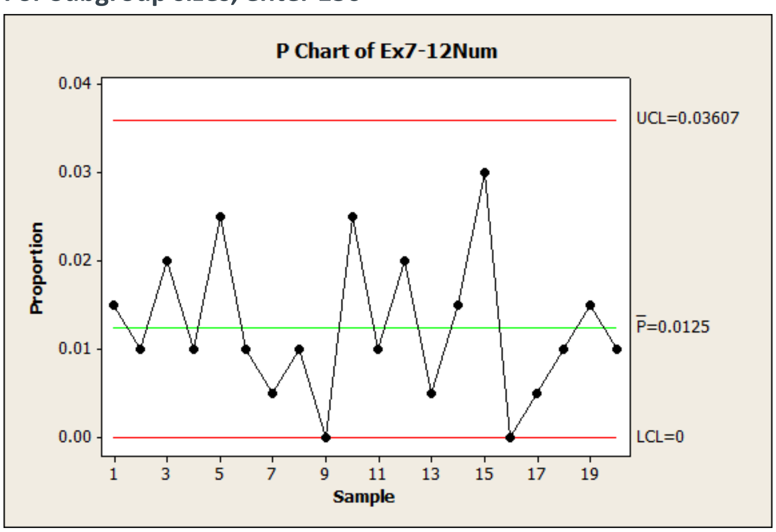
$$n = 200;\ \ m = 20;\ \ \sum_{i=1}^{m} D_i = 50;\ \ \bar{p} = \sum_{i=1}^{m} D_i \Big/ mn = 50/20(200) = 0.0125$$

$$\text{UCL} = \bar{p} + 3\sqrt{\bar{p}(1-\bar{p})/n} = 0.0125 + 3\sqrt{0.0125(1-0.0125)/200} = 0.0361$$

$$\text{LCL} = \bar{p} - 3\sqrt{\bar{p}(1-\bar{p})/n} = 0.0125 - 3\sqrt{0.0125(1-0.0125)/200} = -0.0111 \Rightarrow 0$$

MTB > Stat > Control Charts > Attributes Charts > P

For Subgroup sizes, enter 150



The process appears to be in statistical control.

7.12. continued

(b) What is the smallest sample size that could be used for this process and still give a positive lower control limit on the chart?

Using Equation 7.12,

$$n > \frac{(1-p)}{p}L^2$$

$$> \frac{(1-0.0125)}{0.0125}(3)^2$$

$$> 711 \quad \text{Select } n = 712.$$

7.13.

A process produces rubber belts in lots of size 2500. Inspection records on the last 20 lots reveal the data in Table 7E.7.

TABLE 7E.7
Inspection Data for Exercise 7.13

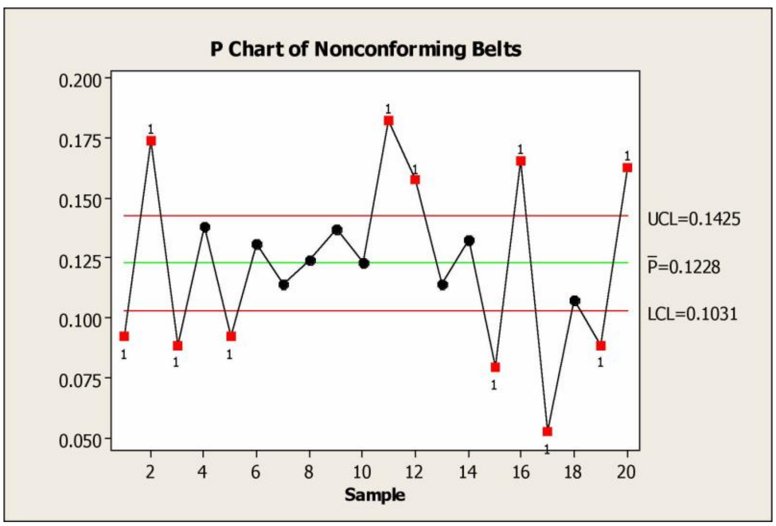
Lot Number	Number of Nonconforming Belts	Lot Number	Number of Nonconforming Belts
1	230	11	456
2	435	12	394
3	221	13	285
4	346	14	331
5	230	15	198
6	327	16	414
7	285	17	131
8	311	18	269
9	342	19	221
10	308	20	407

(a) Compute trial control limits for a fraction nonconforming control chart.

$$\text{UCL} = \bar{p} + 3\sqrt{\bar{p}(1-\bar{p})/n} = 0.1228 + 3\sqrt{0.1228(1-0.1228)/2500} = 0.1425$$

$$\text{LCL} = \bar{p} - 3\sqrt{\bar{p}(1-\bar{p})/n} = 0.1228 - 3\sqrt{0.1228(1-0.1228)/2500} = 0.1031$$

MTB > Stat > Control Charts > Attributes Charts > P
For Subgroup sizes, enter 2500



Test Results for P Chart of Ex7.5Num

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  1, 2, 3, 5, 11, 12, 15, 16, 17, 19, 20
```

(b) If you wanted to set up a control chart for controlling future production, how would you use these data to obtain the center line and control limits for the chart?

So many subgroups are out of control (11 of 20) that the data should not be used to establish control limits for future production. Instead, the process should be investigated for causes of the wild swings in *p*.

7.14.

Based on the data in Table 7E.8 if an *np* chart is to be established, what would you recommend as the center line and control limits? Assume that *n* = 700.

TABLE 7E.8
Data for Exercise 7.14

Day	Number of Nonconforming Units
1	3
2	4
3	3
4	2
5	6
6	12
7	5
8	1
9	2
10	2

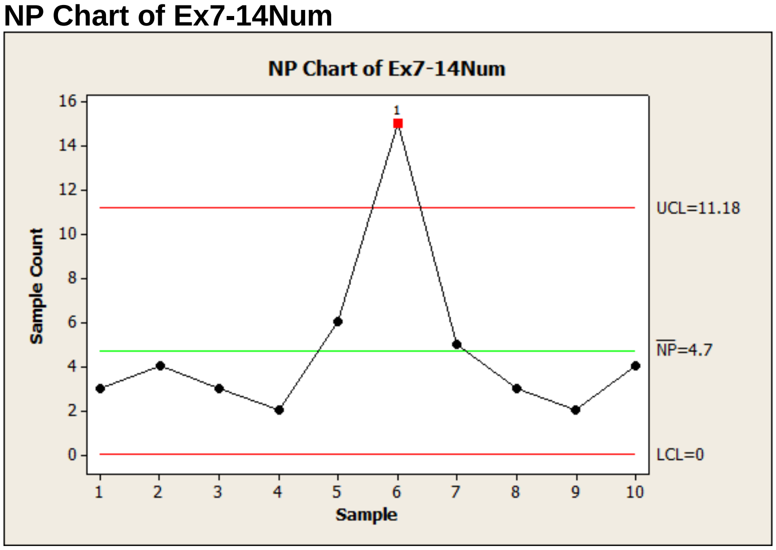
$$n\bar{p} = \sum_{i=1}^{m} D_i \Big/ m = 47/10 = 4.7; \quad \bar{p} = \sum_{i=1}^{m} D_i \Big/ (mn) = 47/(10 \times 700) = 0.0067$$

$$\text{UCL} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} = 4.7 + 3\sqrt{4.7(1-0.0067)} = 11.2$$

$$\text{LCL} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})} = 4.7 - 3\sqrt{4.7(1-0.0067)} = -1.8 \Rightarrow 0$$

MTB > Stat > Control Charts > Attributes Charts > NP
For Subgroup sizes, enter 700

NP Chart of Ex7-14Num



Test Results for NP Chart of Ex7-14Num

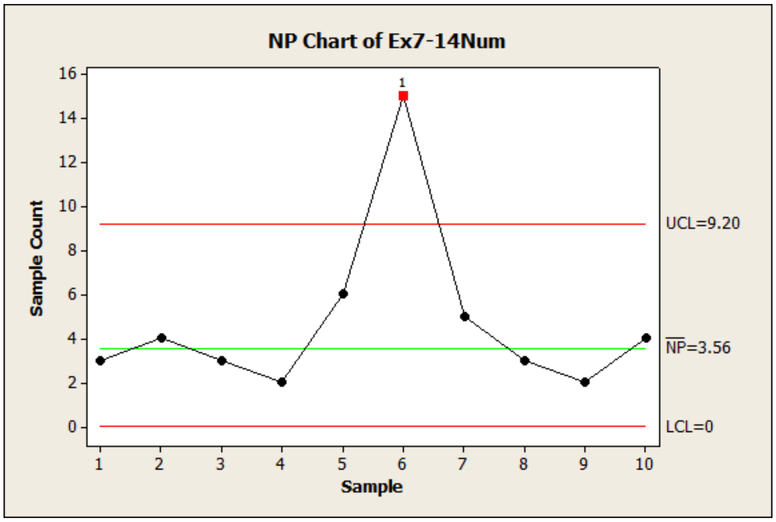
```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  6
```

7.14. continued

Recalculate control limits without sample 6:

MTB > Stat > Control Charts > Attributes Charts > NP
For Subgroup sizes, enter 700
To exclude Sample 6, select NP Chart Options, Estimate tab, and Omit subgroup 6

NP Chart of Ex7-14Num



Test Results for NP Chart of Ex7-14Num

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  6
```

Recommend using control limits from second chart (calculated less sample 6), with center line = 3.56 and UCL = 9.20.

7.15.

A control chart indicates that the current process fraction nonconforming is 0.02. If 50 items are inspected each day, what is the probability of detecting a shift in the fraction nonconforming to 0.04 on the first day after the shift? By the end of the third day following the shift?

$$\bar{p}=0.02; n=50$$
$$\text{UCL}=\bar{p}+3\sqrt{\bar{p}(1-\bar{p})/n}=0.02+3\sqrt{0.02(1-0.02)/50}=0.0794$$
$$\text{LCL}=\bar{p}-3\sqrt{\bar{p}(1-\bar{p})/n}=0.02-3\sqrt{0.02(1-0.02)/50}=0.02-0.0594\Rightarrow 0$$

Since p_{new} = 0.04 < 0.1 and n = 50 is "large", use the Poisson approximation to the binomial with $\lambda = np_{\text{new}}$ = 50(0.04) = 2.00.

Pr{detect|shift}

= 1 – Pr{not detect|shift}

= 1 – β

= 1 – [Pr{D < nUCL | λ} – Pr{$D \leq n$LCL | λ}]

= 1 – Pr{D < 50(0.0794) | 2} + Pr{$D \leq$ 50(0) | 2}

= 1 – POI(3,2) + POI(0,2) = 1 – 0.857 + 0.135 = 0.278

where POI(·) is the cumulative Poisson distribution.

Pr{detected by 3rd sample} = 1 – Pr{detected after 3rd} = 1 – (1 – 0.278)^3 = 0.624

7.16.

A company purchases a small conduit strap in containers of 4,000 each. Ten containers have arrived at the unloading facility, and 500 brackets are selected at random from each container. The fraction nonconforming in each sample are 0, 0, 0, 0.004, 0.008, 0.020, 0.004, 0, 0, and 0.008. Do the data from this shipment indicate statistical control?

(Note: earlier printings of the 7th edition of the textbook indicated a random sample of 200 brackets, not 500.)

$$m=10;\; n=500;\; \sum_{i=1}^{10}\hat{p}_i=0.0440;\; \bar{p}=\frac{0.0440}{10}=0.0044$$
$$\text{UCL}=\bar{p}+3\sqrt{\bar{p}(1-\bar{p})/n}=0.0044+3\sqrt{0.0044(1-0.0044)/500}=0.0133$$
$$\text{UCL}=\bar{p}-3\sqrt{\bar{p}(1-\bar{p})/n}=0.0044-3\sqrt{0.0044(1-0.0044)/500}=-0.0045\Rightarrow 0$$

No. The data from the shipment do not indicate statistical control. The 6th sample exceeds the UCL, $(\hat{p}_6=0.020)>0.0133$.

7.17.

Diodes used on printed circuit boards are produced in lots of size 1000. We wish to control the process producing these diodes by taking samples of size 64 from each lot. If the nominal value of the fraction nonconforming is p = 0.10, determine the parameters of the appropriate control chart. To what level must the fraction nonconforming increase to make the β-risk equal to 0.50? What is the minimum sample size that would give a positive lower control limit for this chart?

$$\bar{p} = 0.10; n = 64$$
$$\text{UCL} = \bar{p} + 3\sqrt{\bar{p}(1-\bar{p})/n} = 0.10 + 3\sqrt{0.10(1-0.10)/64} = 0.2125$$
$$\text{LCL} = \bar{p} - 3\sqrt{\bar{p}(1-\bar{p})/n} = 0.10 - 3\sqrt{0.10(1-0.10)/64} = 0.10 - 0.1125 \Rightarrow 0$$

$$\begin{aligned}\beta &= \Pr\{D < n\text{UCL} \mid p\} - \Pr\{D \le n\text{LCL} \mid p\} \\ &= \Pr\{D < 64(0.2125) \mid p\} - \Pr\{D \le 64(0) \mid p\} \\ &= \Pr\{D < 13.6) \mid p\} - \Pr\{D \le 0 \mid p\}\end{aligned}$$

p	**Pr{D ≤ 13|p}**	**Pr{D ≤ 0|p}**	**β**
0.05	0.999999	0.037524	0.962475
0.10	0.996172	0.001179	0.994993
0.20	0.598077	0.000000	0.598077
0.21	**0.519279**	**0.000000**	**0.519279**
0.22	0.44154	0.000000	0.44154
0.215	0.480098	0.000000	0.480098
0.212	0.503553	0.000000	0.503553

Assuming L = 3 sigma control limits,

$$\begin{aligned}n &> \frac{(1-p)}{p}L^2 \\ &> \frac{(1-0.10)}{0.10}(3)^2 \\ &> 81\end{aligned}$$

7.18.

A control chart for the number of nonconforming connecting rods is maintained on a forging process with *np* = 10.0. A sample of size 150 is taken each day and analyzed.

$np = 10.0;\ n = 150;\ \bar{p} = 10/150 = 0.067$

$\text{UCL} = np + 3\sqrt{np(1-\bar{p})} = 10 + 3\sqrt{10(1-0.067)} = 19.2$

$\text{LCL} = np - 3\sqrt{np(1-\bar{p})} = 10 - 3\sqrt{10(1-0.067)} = 0.8$

(a) What is the probability that a shift in the process average to *np* = 12.0 will be detected on the first day following the shift? What is the probability that the shift will be detected by at least the end of the third day?

Since $\bar{p}_{new} = 12/150 = 0.080$ is greater than 1/(*n* + 1) = 1/(150+1) = 0.007 and less than *n*/(*n* + 1) = 150/(150+1) = 0.993, the normal approximation to the binomial is adequate (Section 3.5.3).

$$\begin{aligned}
\Pr\{\text{detect shift on 1st sample}\} &= 1-\beta \\
&= 1-[\Pr\{D<\text{UCL}\,|\,p\}-\Pr\{D\le\text{LCL}\,|\,p\}] \\
&= 1-\Phi\left(\frac{\text{UCL}+1/2-np}{\sqrt{np(1-p)}}\right)+\Phi\left(\frac{\text{LCL}-1/2-np}{\sqrt{np(1-p)}}\right) \\
&= 1-\Phi\left(\frac{19.2+0.5-12}{\sqrt{12(1-0.080)}}\right)+\Phi\left(\frac{0.8-0.5-12}{\sqrt{12(1-0.080)}}\right) \\
&= 1-\Phi(2.32)+\Phi(-3.52) \\
&= 1-0.9898+0.0002 \\
&= 0.0104
\end{aligned}$$

Pr{detect by at least 3rd}

= 1 – Pr{detected after 3rd}

= 1 – $(1 - 0.0104)^3$

= 0.0309

(b) Find the smallest sample size that will give a positive lower control limit.

Assuming *L* = 3 sigma control limits,

$$\begin{aligned}
n &> \frac{(1-p)}{p}L^2 \\
&> \frac{(1-0.067)}{0.067}(3)^2 \\
&> 125.3
\end{aligned}$$

So, *n* = 126 is the minimum sample size for a positive LCL.

7.19.

A control chart for the fraction nonconforming is to be established using a center line of p = 0.10. What sample size is required if we wish to detect a shift in the process fraction nonconforming to 0.20 with probability 0.50?

$p = 0.10$; $p_{\text{new}} = 0.20$; desire $\Pr\{\text{detect}\} = 0.50$; assume $k = 3$ sigma control limits

$\delta = p_{\text{new}} - p = 0.20 - 0.10 = 0.10$

$$n = \left(\frac{k}{\delta}\right)^2 p(1-p) = \left(\frac{3}{0.10}\right)^2 (0.10)(1-0.10) = 81$$

7.20.

A process is controlled with a fraction nonconforming control chart with three-sigma limits, n = 100, UCL = 0.161, center line = 0.080, and LCL = 0.

(a) Find the equivalent control chart for the number nonconforming.

$np = 100(0.080) = 8$

$\text{UCL} = np + 3\sqrt{np(1-p)} = 8 + 3\sqrt{8(1-0.080)} = 16.14$

$\text{LCL} = np - 3\sqrt{np(1-p)} = 8 - 3\sqrt{8(1-0.080)} = 8 - 8.1388 \Rightarrow 0$

(b) Use the Poisson approximation to the binomial to find the probability of a type I error.

p = 0.080 < 0.1 and n =100 is large, so use Poisson approximation to the binomial.

Pr{type I error} = α

= Pr{D < LCL | p} + Pr{D > UCL | p}

= Pr{D < LCL | p} + [1 – Pr{$D \leq$ UCL | p}]

= Pr{D < 0 | 8} + [1 – Pr{$D \leq$ 16 | 8}]

= 0 + [1 – POI(16,8)]

= 0 + [1 – 0.996]

= 0.004

where POI(·) is the cumulative Poisson distribution.

7.20.(c) continued

(c) Use the correct approximation to find the probability of a type II error if the process fraction nonconforming shifts to 0.2.

np_{new} = 100(0.20) = 20 > 15, so use the normal approximation to the binomial.

$$\begin{aligned}
\Pr\{\text{type II error}\} = \beta \\
&= \Pr\{\hat{p} < \text{UCL} \mid p_{\text{new}}\} - \Pr\{\hat{p} \leq \text{LCL} \mid p_{\text{new}}\} \\
&= \Phi\left(\frac{\text{UCL} - p_{\text{new}}}{\sqrt{p(1-p)/n}}\right) - \Phi\left(\frac{\text{LCL} - p_{\text{new}}}{\sqrt{p(1-p)/n}}\right) \\
&= \Phi\left(\frac{0.161 - 0.20}{\sqrt{0.08(1-0.08)/100}}\right) - \Phi\left(\frac{0 - 0.20}{\sqrt{0.08(1-0.08)/100}}\right) \\
&= \Phi(-1.44) - \Phi(-7.37) \\
&= 0.07494 - 0 \\
&= 0.07494
\end{aligned}$$

(d) What is the probability of detecting the shift in part (c) by at most the fourth sample after the shift?

Pr{detect shift by at most 4th sample}

= 1 – Pr{not detect by 4th}

= 1 – $(0.07494)^4$

= 0.99997

7.21.

A process is being controlled with a fraction nonconforming control chart. The process average has been shown to be 0.07. Three-sigma control limits are used, and the procedure calls for taking daily samples of 400 items.

(a) Calculate the upper and lower control limits.

$\bar{p} = 0.07$; $k = 3$ sigma control limits; $n = 400$

$$\text{UCL} = \bar{p} + 3\sqrt{p(1-p)/n} = 0.07 + 3\sqrt{0.07(1-0.07)/400} = 0.108$$

$$\text{LCL} = \bar{p} - 3\sqrt{p(1-p)/n} = 0.07 - 3\sqrt{0.07(1-0.07)/400} = 0.032$$

(b) If the process average should suddenly shift to 0.10, what is the probability that the shift would be detected on the first subsequent sample?

np_{new} = 400(0.10) = > 40, so use the normal approximation to the binomial.

$$\begin{aligned}
\Pr\{\text{detect on 1st sample}\} &= 1 - \Pr\{\text{not detect on 1st sample}\} \\
&= 1 - \beta \\
&= 1 - [\Pr\{\hat{p} < \text{UCL} \mid p\} - \Pr\{\hat{p} \le \text{LCL} \mid p\}] \\
&= 1 - \Phi\left(\frac{\text{UCL} - p}{\sqrt{p(1-p)/n}}\right) + \Phi\left(\frac{\text{LCL} - p}{\sqrt{p(1-p)/n}}\right) \\
&= 1 - \Phi\left(\frac{0.108 - 0.1}{\sqrt{0.1(1-0.1)/400}}\right) + \Phi\left(\frac{0.032 - 0.1}{\sqrt{0.1(1-0.1)/400}}\right) \\
&= 1 - \Phi(0.533) + \Phi(-4.533) \\
&= 1 - 0.703 + 0.000 \\
&= 0.297
\end{aligned}$$

(c) What is the probability that the shift in part (b) would be detect on the first or second sample taken after the shift?

Pr{detect on 1st or 2nd sample}

= Pr{detect on 1st} + Pr{not on 1st}×Pr{detect on 2nd}

= 0.297 + (1 – 0.297)(0.297)

= 0.506

7.22.

In designing a fraction nonconforming chart with center line at p = 0.20 and three-sigma control limits, what is the sample size required to yield a positive lower control limit? What is the value of n necessary to give a probability of 0.50 of detecting a shift in the process to 0.26?

p = 0.20 and L = 3 sigma control limits

$$n > \frac{(1-p)}{p}L^2$$
$$> \frac{(1-0.20)}{0.20}(3)^2$$
$$> 36$$

For Pr{detect} = 0.50 after a shift to p_{new} = 0.26,

$$\delta = p_{\text{new}} - p = 0.26 - 0.20 = 0.06$$

$$n = \left(\frac{k}{\delta}\right)^2 p(1-p) = \left(\frac{3}{0.06}\right)^2 (0.20)(1-0.20) = 400$$

7.23.

A control chart is used to control the fraction nonconforming for a plastic part manufactured in an injection molding process. Ten subgroups yield the data in Table 7E.9.

TABLE 7E.9

Nonconforming Unit Data for Exercise 7.23

Sample Number	Sample Size	Number Nonconforming
1	100	10
2	100	15
3	100	31
4	100	18
5	100	24
6	100	12
7	100	23
8	100	15
9	100	8
10	100	8

(a) Set up a control chart for the number nonconforming in samples of n = 100.

$$m = 10; \quad n = 100; \quad \sum_{i=1}^{10} D_i = 164; \quad \bar{p} = \sum_{i=1}^{10} D_i \Big/ (mn) = 164/[10(100)] = 0.164; \quad n\bar{p} = 16.4$$

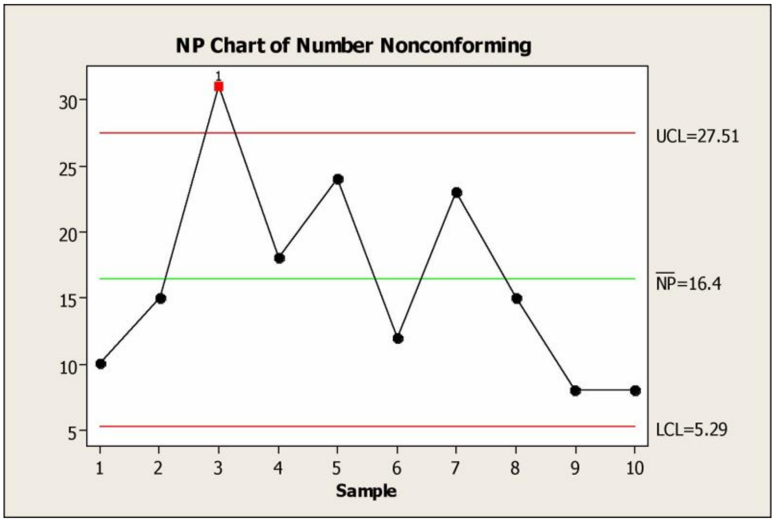
$$\text{UCL} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} = 16.4 + 3\sqrt{16.4(1-0.164)} = 27.51$$

$$\text{LCL} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})} = 16.4 - 3\sqrt{16.4(1-0.164)} = 5.292$$

7.23.(a) continued

MTB > Stat > Control Charts > Attributes Charts > NP

For Subgroup sizes, enter 100



Test Results for NP Chart of Ex7.15Num

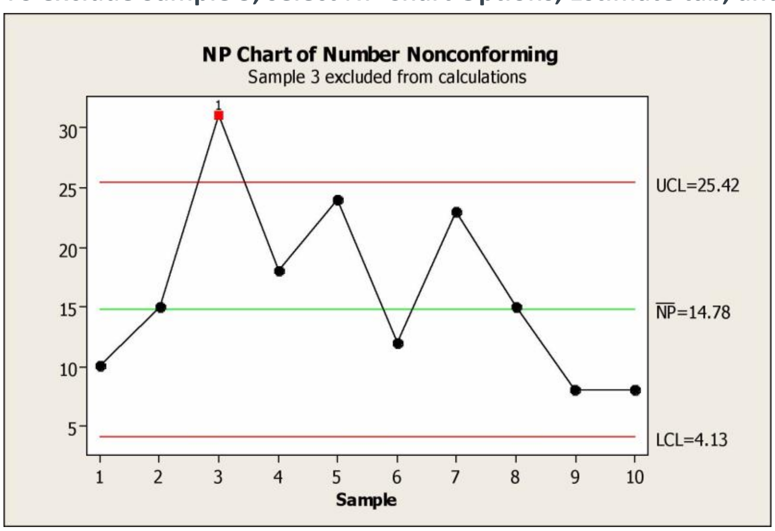
```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  3
```

Recalculate control limits without sample 3:

MTB > Stat > Control Charts > Attributes Charts > NP

For Subgroup sizes, enter 100

To exclude Sample 3, select NP Chart Options, Estimate tab, and Omit subgroup 3



Test Results for NP Chart of Ex7.15Num

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  3
```

Recommend using control limits from second chart (calculated less sample 3), with center line = 14.78 and UCL = 25.42.

7.23. continued

(b) For the chart established in part (a), what is the probability of detecting a shift in the process fraction nonconforming to 0.30 on the first sample after the shift has occurred?

$p_{\text{new}} = 0.30$. Since $p = 0.30$ is not too far from 0.50, and $n = 100 > 10$, the normal approximation to the binomial can be used.

$$
\begin{aligned}
\Pr\{\text{detect on 1st}\} &= 1 - \Pr\{\text{not detect on 1st}\} \\
&= 1 - \beta \\
&= 1 - [\Pr\{D < \text{UCL} \mid p\} - \Pr\{D \le \text{LCL} \mid p\}] \\
&= 1 - \Phi\left(\frac{\text{UCL} + 1/2 - np}{\sqrt{np(1-p)}}\right) + \Phi\left(\frac{\text{LCL} - 1/2 - np}{\sqrt{np(1-p)}}\right) \\
&= 1 - \Phi\left(\frac{25.42 + 0.5 - 30}{\sqrt{30(1-0.3)}}\right) + \Phi\left(\frac{4.13 - 0.5 - 30}{\sqrt{30(1-0.3)}}\right) \\
&= 1 - \Phi(-0.8903) + \Phi(-5.7544) \\
&= 1 - (0.187) + (0.000) \\
&= 0.813
\end{aligned}
$$

7.24.

A control chart for fraction nonconforming indicates that the current process average is 0.05. The sample size is constant at 250 units.

(a) Find the three-sigma control limits for the control chart.

$$\text{UCL}_p = \bar{p} + 3\sqrt{\bar{p}(1-\bar{p})/n} = 0.05 + 3\sqrt{0.05(1-0.05)/250} = 0.0914$$

$$\text{LCL}_p = \bar{p} - 3\sqrt{\bar{p}(1-\bar{p})/n} = 0.05 - 3\sqrt{0.05(1-0.05)/250} = 0.0086$$

(b) What is the probability that a shift in the process average to 0.09 will be detected on the first subsequent sample? What is the probability that this shift will be detected at least by the fourth sample following the shift?.

$p_{\text{new}} = 0.09$. Since ($p_{\text{new}} = 0.09$) < 0.10 and n is large, use the Poisson approximation to the binomial.

$$
\begin{aligned}
\Pr\{\text{detect on 1st sample} \mid p\} &= 1 - \Pr\{\text{not detect} \mid p\} \\
&= 1 - \beta \\
&= 1 - [\Pr\{\hat{p} < \text{UCL} \mid p\} - \Pr\{\hat{p} \le \text{LCL} \mid p\}] \\
&= 1 - \Pr\{D < n\text{UCL} \mid np\} + \Pr\{D \le n\text{LCL} \mid np\} \\
&= 1 - \Pr\{D < 250(0.0914) \mid 250(0.09)\} + \Pr\{D \le 250(0.0086) \mid 250(0.09)\} \\
&= 1 - \Pr\{D < 22.85 \mid 22.5\} + \Pr\{D \le 2.15 \mid 22.5\} \\
&= 1 - \text{POI}(22, 22.5) + \text{POI}(2, 22.5) \\
&= 1 - 0.5141 + 0.000 \\
&= 0.4859
\end{aligned}
$$

where POI(·) is the cumulative Poisson distribution.

Pr{detect by at least 4th} = 1 – Pr{detect after 4th} = 1 – $(1 - 0.4859)^4$ = 0.9301

7.25.

(a)

A control chart for the number nonconforming is to be established, based on samples of size 400. To start the control chart, 30 samples were selected and the number nonconforming in each sample determined, yielding $\sum_{i=1}^{30} D_i = 1{,}200$. What are the parameters of the *np* chart?

$$\bar{p} = \sum_{i=1}^{m} D_i \Big/ (mn) = 1200 \big/ [30(400)] = 0.10; \quad n\bar{p} = 400(0.10) = 40$$

$$\text{UCL}_{np} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} = 40 + 3\sqrt{40(1-0.10)} = 58$$

$$\text{LCL}_{np} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})} = 40 - 3\sqrt{40(1-0.10)} = 22$$

(b) Suppose the process average fraction nonconforming shifted to 0.15. What is the probability that the shift would be detected on the first subsequent sample?

np_{new} = 400 (0.15) = 60 > 15, so use the normal approximation to the binomial.

$$\begin{aligned}
\Pr\{\text{detect on 1st sample} \mid p\} &= 1 - \Pr\{\text{not detect on 1st sample} \mid p\} \\
&= 1 - \beta \\
&= 1 - [\Pr\{D < \text{UCL} \mid np\} - \Pr\{D \le \text{LCL} \mid np\}] \\
&= 1 - \Phi\left(\frac{\text{UCL} + 1/2 - np}{\sqrt{np(1-p)}}\right) + \Phi\left(\frac{\text{LCL} - 1/2 - np}{\sqrt{np(1-p)}}\right) \\
&= 1 - \Phi\left(\frac{58 + 0.5 - 60}{\sqrt{60(1-0.15)}}\right) + \Phi\left(\frac{22 - 0.5 - 60}{\sqrt{60(1-0.15)}}\right) \\
&= 1 - \Phi(-0.210) + \Phi(-5.39) \\
&= 1 - 0.417 + 0.000 \\
&= 0.583
\end{aligned}$$

7.26.

A fraction nonconforming control chart with center line 0.20, UCL = 0.32, and LCL = 0.08 is used to control a process.

(a) If three-sigma limits are used, find the sample size for the control chart.

$$\text{UCL} = p + 3\sqrt{p(1-p)/n}$$

$$n = p(1-p)\left(\frac{3}{\text{UCL}-p}\right)^2 = 0.2(1-0.2)\left(\frac{3}{0.32-0.2}\right)^2 = 100$$

(b) Use the Poisson approximation to the binomial to find the probability of type I error.

Using the Poisson approximation to the binomial, $\lambda = np = 100(0.20) = 20$

$$\begin{aligned}
\Pr\{\text{type I error}\} &= \Pr\{\hat{p} < \text{LCL} \mid p\} + \Pr\{\hat{p} > \text{UCL} \mid p\} \\
&= \Pr\{D < n\text{LCL} \mid \lambda\} + 1 - \Pr\{D \le n\text{UCL} \mid \lambda\} \\
&= \Pr\{D < 100(0.08) \mid 20\} + 1 - \Pr\{D \le 100(0.32) \mid 20\} \\
&= \Pr\{D < 8 \mid 20\} + 1 - \Pr\{D \le 32 \mid 20\} \\
&= \text{POI}(7,20) + 1 - \text{POI}(32,20) \\
&= 0.0008 + 1 - 0.9953 \\
&= 0.0055
\end{aligned}$$

where POI(·) is the cumulative Poisson distribution.

(c) Use the Poisson approximation to the binomial to find the probability of type II error if the process fraction defective is actually $p = 0.40$.

$p_{\text{new}} = 0.40$

Using the Poisson approximation to the binomial, $\lambda = np_{\text{new}} = 100(0.40) = 40$

$$\begin{aligned}
\Pr\{\text{type II error}\} &= \beta \\
&= \Pr\{D < n\text{UCL} \mid \lambda\} - \Pr\{D \le n\text{LCL} \mid \lambda\} \\
&= \Pr\{D < 100(0.32) \mid 40\} - \Pr\{D \le 100(0.08) \mid 40\} \\
&= \Pr\{D < 32 \mid 40\} - \Pr\{D \le 8 \mid 40\} \\
&= \text{POI}(31,40) - \text{POI}(8,40) \\
&= 0.0855 - 0.0000 \\
&= 0.0855
\end{aligned}$$

where POI(·) is the cumulative Poisson distribution.

7.27.

Consider the control chart designed in Exercise 7.25. Find the average run length to detect a shift to a fraction nonconforming of 0.15.

from 7.25.(b), $1 - \beta = 0.583$

$ARL_1 = 1/(1 - \beta) = 1/(0.583) = 1.715 \cong 2$

7.28.

Consider the control chart in Exercise 7.26. Find the average run length if the process fraction nonconforming shifts to 0.40.

from 7.26(c), $\beta = 0.0855$

$ARL_1 = 1/(1 - \beta) = 1/(1 - 0.0855) = 1.1 \cong 1$

7.29.

A maintenance group improves the effectiveness of its repair work by monitoring the number of maintenance requests that require a second call to complete the repair. Twenty weeks of data are shown in Table 7E.10.

TABLE 7E.10

Data for Exercise 7.29

Week	Total Requests	Second Visit Required	Week	Total Requests	Second Visit Required
1	200	6	11	100	1
2	250	8	12	100	0
3	250	9	13	100	1
4	250	7	14	200	4
5	200	3	15	200	5
6	200	4	16	200	3
7	150	2	17	200	10
8	150	1	18	200	4
9	150	0	19	250	7
10	150	2	20	250	6

(a) Find trial control limits for this process.

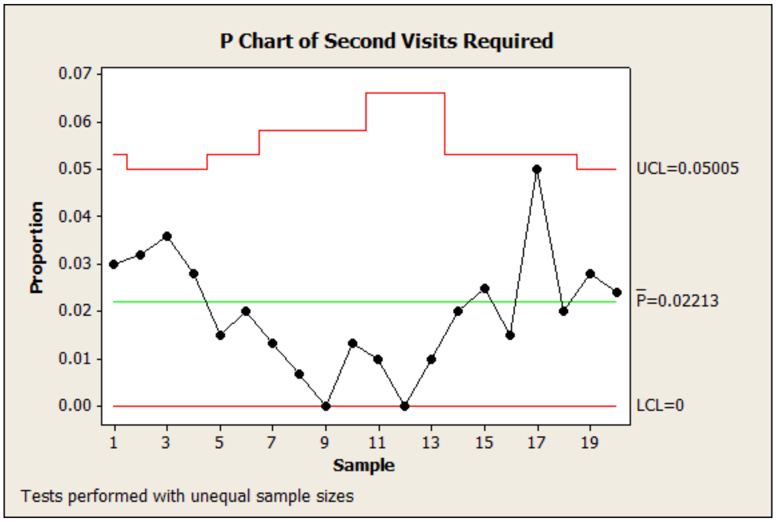
For a *p* chart with variable sample size: $\bar{p} = \sum_i D_i / \sum_i n_i = 83/3750 = 0.0221$ and control limits are at

$\bar{p} \pm 3\sqrt{\bar{p}(1-\bar{p})/n_i}$

n_i	$[LCL_i, UCL_i]$
100	[0, 0.0662]
150	[0, 0.0581]
200	[0, 0.0533]
250	[0, 0.0500]

7.29.(a) continued

MTB > Stat > Control Charts > Attributes Charts > P
For Subgroup sizes, enter column with Total Requests



Process is in statistical control.

(b) Design a control chart for controlling future production.

There are two approaches for controlling future production. The first approach would be to plot $\hat{p}_i$ and use constant limits unless there is a different size sample or a plot point near a control limit. In those cases, calculate the exact control limits by $\bar{p} \pm 3\sqrt{\bar{p}(1-\bar{p})/n_i} = 0.0221 \pm 3\sqrt{0.0216/n_i}$. The second approach, preferred in many cases, would be to construct standardized control limits with control limits at $\pm$ 3, and to plot $Z_i = (\hat{p}_i - 0.0221)\Big/\sqrt{0.0221(1-0.0221)/n_i}$.

7.30.

Analyze the data in Exercise 7.29 using an average sample size.

MTB > Stat > Basic Statistics > Display Descriptive Statistics

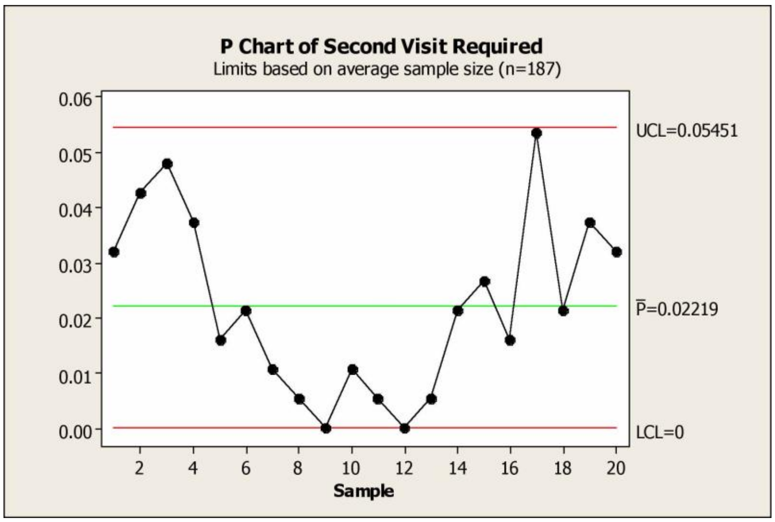
Descriptive Statistics: Ex7.21Req

```
Variable    N   Mean
Ex7.21Req  20  187.5
```

Average sample size is 187.5, however Minitab accepts only integer values for *n*. Use a sample size of *n* = 187, and carefully examine points near the control limits.

MTB > Stat > Control Charts > Attributes Charts > P

For Subgroup sizes, enter average size of 187



Process is in statistical control.

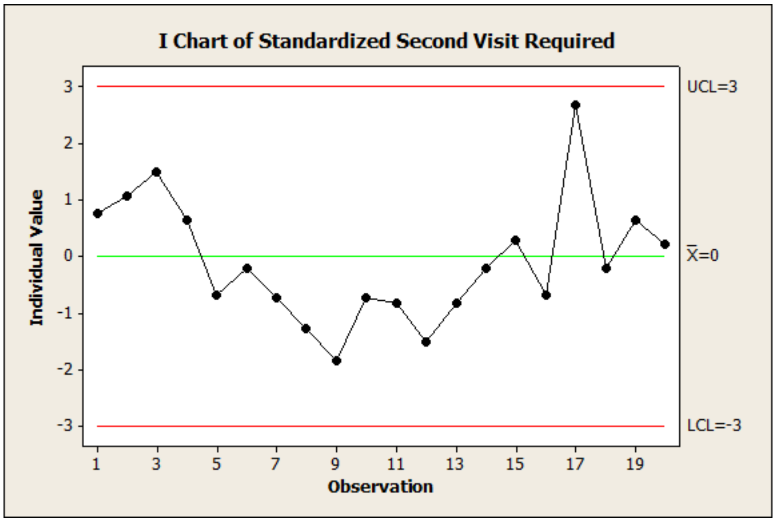
7.31.

Construct a standardized control chart for the data in Exercise 7.29.

$$z_i = (\hat{p}_i - \bar{p}) \Big/ \sqrt{\bar{p}(1-\bar{p})/n_i} = (\hat{p}_i - 0.0221) \Big/ \sqrt{0.0216 / n_i}$$

MTB > Stat > Control Charts > Variables Charts for Individuals > Individuals

Under I Chart Options, on the Parameters tab, enter Mean = 0 and Standard deviation = 1



Process is in statistical control.

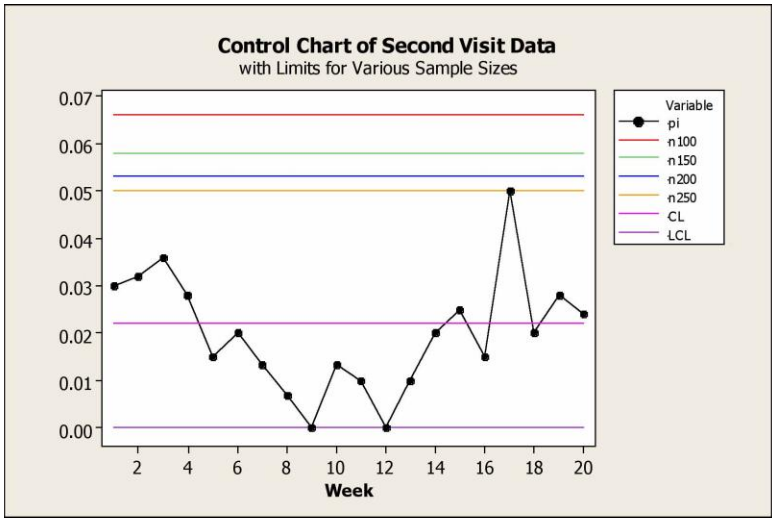
7.32.

Continuation of Exercise 7.29. Note that in Exercise 7.29 there are only four different sample sizes; n = 100, 150, 200, and 250. Prepare a control chart that has a set of limits for each possible sample size and show how it could be used as an alternative to the variable-width control limit method used in Exercise 7.29. How easy would this method be to use in practice?

CL = 0.0221, LCL = 0

UCL_{100} = 0.0662, UCL_{150} = 0.0581, UCL_{200} = 0.0533, UCL_{250} = 0.0500

MTB > Graph > Time Series Plot > Multiple



The difficulty in using this chart in practice would be referencing the appropriate UCL for each sample. One could also color code the plot points, but to this user the variable UCL is easier to interpret.

7.33. ☺

A process has an in-control fraction nonconforming of p = 0.02. What sample size would be required for the fraction nonconforming control chart if it is desired to have a probability of at least one nonconforming unit in the sample to be at least 0.95?

Find n such that $Pr\{D \geq 1\} \geq 0.95$, or equivalently $Pr\{D = 0\} = 0.05$ (text page 308)

From the binomial distribution, for p = 0.02:

$$\Pr\{D=0\} = \frac{n!}{0!(n-0)!}(0.02)^0(1-0.02)^{n-0}$$

$$0.05 = 0.98^n$$

$$n = \frac{\ln(0.05)}{\ln(0.98)} \approx 149$$

7.34. ☺

A process has an in-control fraction nonconforming of $p = 0.02$. What sample size would be required for the fraction nonconforming control chart if it is desired to have a probability of at least one nonconforming unit in the sample to be at least 0.95?

Find n such that $Pr\{D \geq 1\} \geq 0.95$, or equivalently $Pr\{D = 0\} = 0.1$ (text page 308)

From the binomial distribution, for $p = 0.02$:

$$\Pr\{D=0\} = \frac{n!}{0!(n-0)!}(0.02)^0(1-0.02)^{n-0}$$

$$0.2 = 0.98^n$$

$$n = \frac{\ln(0.2)}{\ln(0.98)} \approx 80$$

7.35. ☺

A process has an in-control fraction nonconforming of $p = 0.01$. The sample size is $n = 300$. What is the probability of detecting a shift to an out-of-control fraction nonconforming of $p = 0.05$ on the first sample following the shift?

$$\bar{p} = 0.01; n = 300$$

$$\text{UCL} = \bar{p} + 3\sqrt{\bar{p}(1-\bar{p})/n} = 0.01 + 3\sqrt{0.01(1-0.01)/300} = 0.01 + 0.0172 = 0.0272$$

$$\text{LCL} = \bar{p} - 3\sqrt{\bar{p}(1-\bar{p})/n} = 0.01 - 3\sqrt{0.01(1-0.01)/300} = 0.01 - 0.0172 \Rightarrow 0$$

Since $p_{\text{new}} = 0.05 < 0.1$ and $n = 300$ is "large", use the Poisson approximation to the binomial with $\lambda = np_{\text{new}} = 300(0.05) = 15$.

Pr{detect|shift}

= 1 – Pr{not detect|shift}

= $1 - \beta$

= $1 - [\Pr\{D < n\text{UCL} \mid \lambda\} - \Pr\{D \leq n\text{LCL} \mid \lambda\}]$

= $1 - \Pr\{D < 300(0.0272) \mid 15\} + \Pr\{D \leq 300(0) \mid 15\}$

= 1 – POI(8,15) + POI(0,15) = 1 – 0.037 + 0.000 = 0.963

where POI(·) is the cumulative Poisson distribution.

7.36. ☺

A banking center has instituted a process improvement program to reduce and hopefully eliminate errors in their check processing operations. The current error rate is 0.01. The initial objective is to cut the current error rate in half. What sample size would be necessary to monitor this process with a fraction nonconforming control chart that has a non-zero LCL? If the error rate is reduced to the desired initial target of 0.005, what is the probability of a sample nonconforming from this improved process falling below the LCL?

Assuming that three-sigma control limits are used, the minimum sample size needed for a non-zero LCL is:

$$\begin{aligned} n &> \frac{(1-p)}{p}L^2 \\ &> \frac{(1-0.01)}{0.01}3^2 \\ &> 891 \\ &\geq 892 \end{aligned}$$

$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.01 - 3\sqrt{\frac{0.01(1-0.01)}{892}} = 0.01 - 0.00999 = 0.00001$$

Since $p_{\text{new}} = 0.005 < 0.1$ and $n = 892$ is "large", use the Poisson approximation to the binomial with $\lambda = np_{\text{new}} = 892(0.005) = 4.46$.

$$\Pr\{p_i < \text{LCL}\} = \Pr\{D \leq n\text{LCL} \mid \lambda\} = \Pr\{D \leq 892(0.00001) \mid lambda\} = \Pr\{D \leq 0.00892 \mid 4.46\} = \text{POI}(0, 4.46)$$

MTB > Calc > Probability Distributions > Poisson

Cumulative Distribution Function

```
Poisson with mean = 4.46
x  P( X <= x )
0    0.0115624
```

The probability of a nonconforming sample from the improved process falling below the LCL is 0.01.

7.37.

A fraction nonconforming control chart has center line 0.01, UCL = 0.0399, LCL = 0, and n = 100. If three-sigma limits are used, find the smallest sample size that would yield a positive lower control limit.

$$\begin{aligned} n &> \left(\frac{1-p}{p}\right)L^2 \\ &> \left(\frac{1-0.01}{0.01}\right)3^2 \\ &> 891 \\ &\geq 892 \end{aligned}$$

7.38.

Why is the *np* chart not appropriate with variable sample size?

The *np* chart is inappropriate for varying sample sizes because the centerline (process center) would change with each n_i.

7.39.

A fraction nonconforming control chart with n = 400 has the following parameters: UCL = 0.0809, Center line = 0.0500, LCL = 0.0191.

(a) Find the width of the control limits in standard deviation units.

$$0.0809 = 0.05 + L\sqrt{0.05(1-0.05)/400} = 0.05 + L(0.0109)$$
$$L = 2.8349$$

(b) What would be the corresponding parameters for an equivalent control chart based on the number nonconforming?

$$\text{CL} = np = 400(0.05) = 20$$
$$\text{UCL} = np + 2.8349\sqrt{np(1-p)} = 20 + 2.8349\sqrt{20(1-0.05)} = 32.36$$
$$\text{LCL} = np - 2.8349\sqrt{np(1-p)} = 20 - 2.8349\sqrt{20(1-0.05)} = 7.64$$

(c) What is the probability that a shift in the process fraction nonconforming to 0.0300 will be detected on the first sample following the shift?

n = 400 is large and p = 0.05 < 0.1, use Poisson approximation to binomial.

$$\begin{aligned}
&\Pr\{\text{detect shift to 0.03 on 1st sample}\}\\
&= 1 - \Pr\{\text{not detect}\}\\
&= 1 - \beta\\
&= 1 - [\Pr\{D < \text{UCL} \mid \lambda\} - \Pr\{D \le \text{LCL} \mid \lambda\}]\\
&= 1 - \Pr\{D < 32.36 \mid 12\} + \Pr\{D \le 7.64 \mid 12\}\\
&= 1 - \text{POI}(32,12) + \text{POI}(7,12)\\
&= 1 - 1.0000 + 0.0895\\
&= 0.0895
\end{aligned}$$

where POI(·) is the cumulative Poisson distribution.

7.40.
A fraction nonconforming control chart with n = 500 has the following parameters:
UCL = 0.0841; Center line = 0.0500; LCL = 0.0159.

(a) Find the width of the control limits in standard deviation units.

$$\begin{aligned} \text{UCL} &= p + L\sqrt{p(1-p)/n} \\ 0.0841 &= 0.0500 + L\sqrt{0.05(1-0.05)/500} \\ L &= (0.0841-0.0500)\Big/\sqrt{0.05(1-0.05)/500} = 3.5 \end{aligned}$$

(b) Suppose the process fraction nonconforming shifts to 0.12. What is the probability of detecting the shift on the first subsequent sample?

p = 0.12, λ = np = 500(0.12) = 60 > 15, use normal approximation to binomial.

$$\begin{aligned} &\Pr\{\text{detect on 1st sample after shift}\} \\ &= 1 - \Pr\{\text{not detect}\} \\ &= 1 - \beta \\ &= 1 - [\Pr\{\hat{p} < \text{UCL} \mid p\} - \Pr\{\hat{p} \le \text{LCL} \mid p\}] \\ &= 1 - \Phi\left(\frac{\text{UCL}-p}{\sqrt{p(1-p)/n}}\right) + \Phi\left(\frac{\text{LCL}-p}{\sqrt{p(1-p)/n}}\right) \\ &= 1 - \Phi\left(\frac{0.0841-0.12}{\sqrt{0.12(1-0.12)/500}}\right) + \Phi\left(\frac{0.0159-0.12}{\sqrt{0.12(1-0.12)/500}}\right) \\ &= 1 - \Phi(-2.47) + \Phi(-7.18) \\ &= 1 - 0.0068 + 0.0000 \\ &= 0.9932 \end{aligned}$$

7.41.
A fraction nonconforming control chart is to be established with a center line of 0.01 and two-sigma control limits.

(a) How large should the sample size be if the lower control limit is to be nonzero?

$$\begin{aligned} n &> \left(\frac{1-p}{p}\right)L^2 \\ &> \left(\frac{1-0.01}{0.01}\right)2^2 \\ &> 396 \\ &\ge 397 \end{aligned}$$

7.41. continued

(b) How large should the sample size be if we wish the probability of detecting a shift to 0.04 to be 0.50?

$\delta = 0.04 - 0.01 = 0.03$

$$n = \left(\frac{L}{\delta}\right)^2 p(1-p) = \left(\frac{2}{0.03}\right)^2 (0.01)(1-0.01) = 44$$

7.42.

The following fraction nonconforming control chart with n = 100 is used to control a process:
UCL = 0.0750; Center line = 0.0400; LCL = 0.0050

(a) Use the Poisson approximation to the binomial to find the probability of a type I error.

$$\begin{aligned}
&\Pr\{\text{type I error}\} \\
&= \Pr\{\hat{p} < \text{LCL} \mid p\} + \Pr\{\hat{p} > \text{UCL} \mid p\} \\
&= \Pr\{D < n\text{LCL} \mid np\} + 1 - \Pr\{D \le n\text{UCL} \mid np\} \\
&= \Pr\{D < 100(0.0050) \mid 100(0.04)\} + 1 - \Pr\{D \le 100(0.075) \mid 100(0.04)\} \\
&= \text{POI}(0,4) + 1 - \text{POI}(7,4) \\
&= 0.018 + 1 - 0.948 \\
&= 0.070
\end{aligned}$$

where POI(·) is the cumulative Poisson distribution.

(b) Use the Poisson approximation to the binomial to find the probability of a type II error, if the true process fraction nonconforming is 0.0600.

$$\begin{aligned}
&\Pr\{\text{type II error}\} \\
&= \beta \\
&= \Pr\{D < n\text{UCL} \mid np\} - \Pr\{D \le n\text{LCL} \mid np\} \\
&= \Pr\{D < 100(0.075) \mid 100(0.06)\} - \Pr\{D \le 100(0.005) \mid 100(0.06) \\
&= \text{POI}(7,6) - \text{POI}(0,6) \\
&= 0.744 - 0.002 \\
&= 0.742
\end{aligned}$$

where POI(·) is the cumulative Poisson distribution.

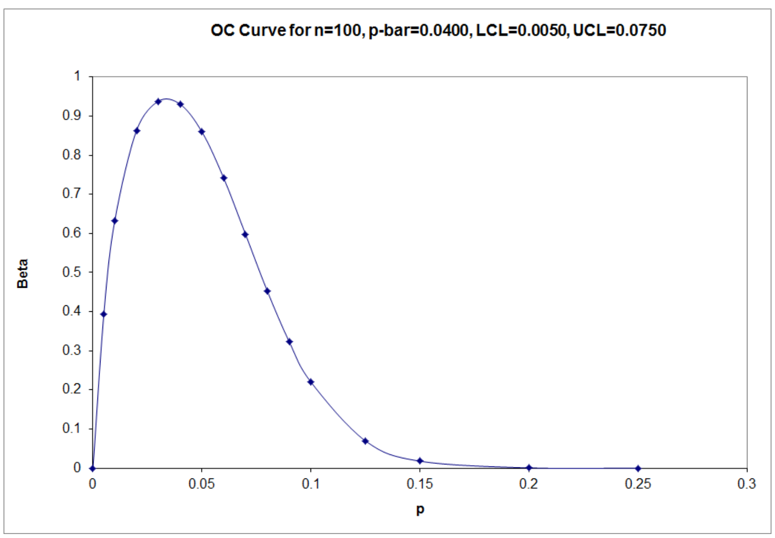
7.42. continued

(c) Draw the OC curve for this control chart.

$$\begin{aligned}\beta &= \Pr\{D < n\text{UCL} \mid np\} - \Pr\{D \le n\text{LCL} \mid np\} \\ &= \Pr\{D < 100(0.0750) \mid 100p\} - \Pr\{D \le 100(0.0050) \mid 100p\} \\ &= \Pr\{D < 7.5 \mid 100p\} - \Pr\{D \le 0.5 \mid 100p\}\end{aligned}$$

Note: The Instructor's data set has the Excel solution for this problem.

p	np	Pr{D<7.5|np}	Pr{D<=0.5|np}	beta
0	0	1.0000	1.0000	0.0000
0.005	0.5	1.0000	0.6065	0.3935
0.01	1	1.0000	0.3679	0.6321
0.02	2	0.9989	0.1353	0.8636
0.03	3	0.9881	0.0498	0.9383
0.04	4	0.9489	0.0183	0.9306
0.05	5	0.8666	0.0067	0.8599
0.06	6	0.7440	0.0025	0.7415
0.07	7	0.5987	0.0009	0.5978
0.08	8	0.4530	0.0003	0.4526
0.09	9	0.3239	0.0001	0.3238
0.1	10	0.2202	0.0000	0.2202
0.125	12.5	0.0698	0.0000	0.0698
0.15	15	0.0180	0.0000	0.0180
0.2	20	0.0008	0.0000	0.0008
0.25	25	0.0000	0.0000	0.0000



(d) Find the ARL when the process is in control and the ARL when the process fraction nonconforming is 0.0600.

from part (a), $\alpha = 0.070$: $\text{ARL}_0 = 1/\alpha = 1/0.070 = 14.29 \cong 15$

from part (b), $\beta = 0.0742$: $\text{ARL}_1 = 1/(1-\beta) = 1/(1-0.742) = 3.861 \cong 4$

7.43.

A process that produces bearing housings is controlled with a fraction nonconforming control chart, using sample size n= 100 and a center line $\overline{p} = 0.02$.

(a) Find the three-sigma limits for this chart.

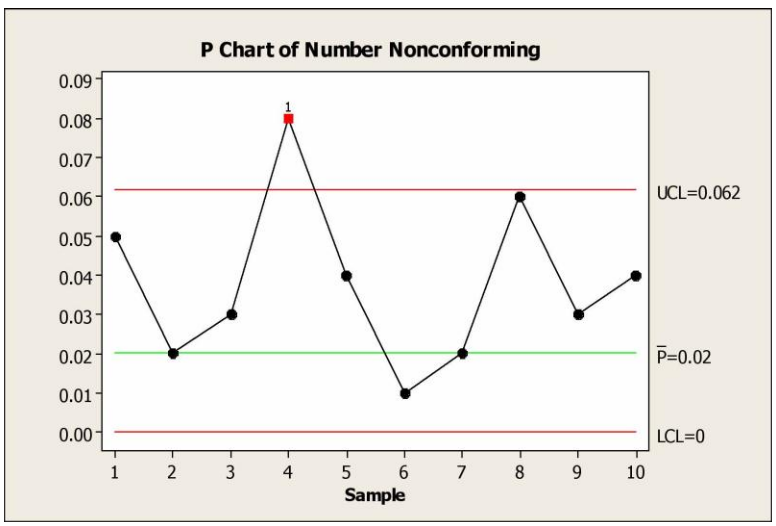
$$\text{UCL} = \overline{p} + 3\sqrt{\overline{p}(1-\overline{p})/n} = 0.02 + 3\sqrt{0.02(1-0.02)/100} = 0.062$$
$$\text{LCL} = \overline{p} - 3\sqrt{\overline{p}(1-\overline{p})/n} = 0.02 - 3\sqrt{0.02(1-0.02)/100} \Rightarrow 0$$

(b) Analyze the ten new samples (n = 100) shown in Table 7E.11 for statistical control. What conclusions can you draw about the process now?

TABLE 7E.11
Data for Exercise 7.43, part (b)

Sample Number	Number Nonconforming	Sample Number	Number Nonconforming
1	5	6	1
2	2	7	2
3	3	8	6
4	8	9	3
5	4	10	4

MTB > Stat > Control Charts > Attributes Charts > P



Test Results for P Chart of Ex7.31Num

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:   4
```

Sample 4 exceeds the upper control limit.

$\overline{p} = 0.038$ and $\hat{\sigma}_p = 0.0191$

7.44.

Consider an *np* chart with *k*-sigma control limits. Derive a general formula for determining the minimum sample size to ensure that the chart has a positive lower control limit.

$$\text{LCL} = n\bar{p} - k\sqrt{n\bar{p}(1-\bar{p})} > 0$$

$$n\bar{p} > k\sqrt{n\bar{p}(1-\bar{p})}$$

$$n > k^2\left(\frac{1-\bar{p}}{\bar{p}}\right)$$

7.45.

Consider the fraction nonconforming control chart in Exercise 7.12. Find the equivalent *np* chart.

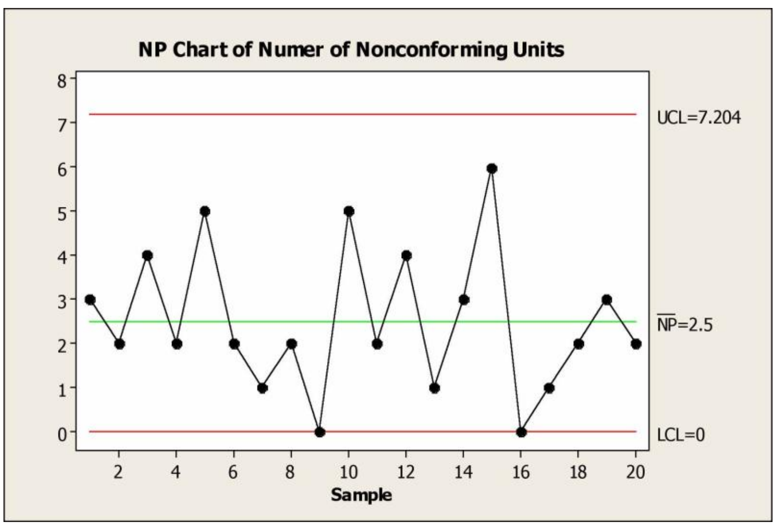
$$n = 150; \quad m = 20; \quad \sum D = 50; \quad \bar{p} = 0.0167$$

$$\text{CL} = n\bar{p} = 150(0.0167) = 2.505$$

$$\text{UCL} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} = 2.505 + 3\sqrt{2.505(1-0.0167)} = 7.213$$

$$\text{LCL} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})} = 2.505 - 4.708 \Rightarrow 0$$

MTB > Stat > Control Charts > Attributes Charts > NP



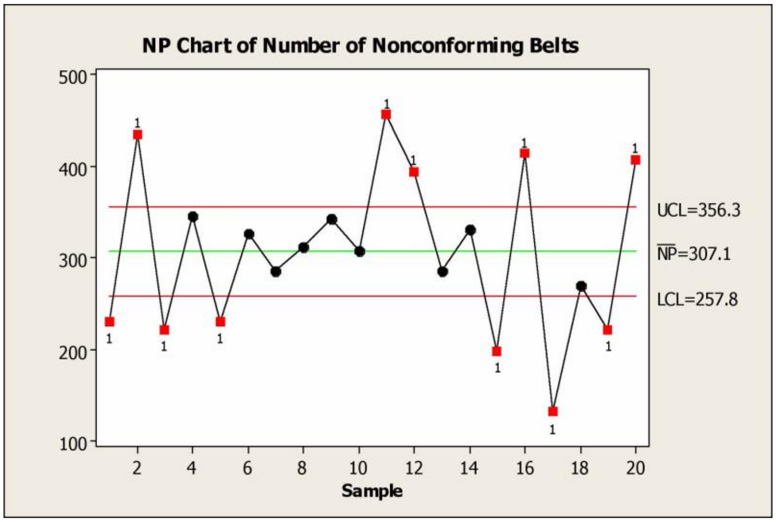
The process is in control; results are the same as for the *p* chart.

7.46.

Consider the fraction nonconforming control chart in Exercise 7.13. Find the equivalent *np* chart.

$$\text{CL} = n\bar{p} = 2500(0.1228) = 307$$
$$\text{UCL} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} = 307 + 3\sqrt{307(1-0.1228)} = 356.23$$
$$\text{LCL} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})} = 307 - 3\sqrt{307(1-0.1228)} = 257.77$$

MTB > Stat > Control Charts > Attributes Charts > NP



Test Results for NP Chart of Ex7.5Num

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  1, 2, 3, 5, 11, 12, 15, 16, 17, 19, 20
```

Like the *p* control chart, many subgroups are out of control (11 of 20), indicating that this data should not be used to establish control limits for future production.

7.47.

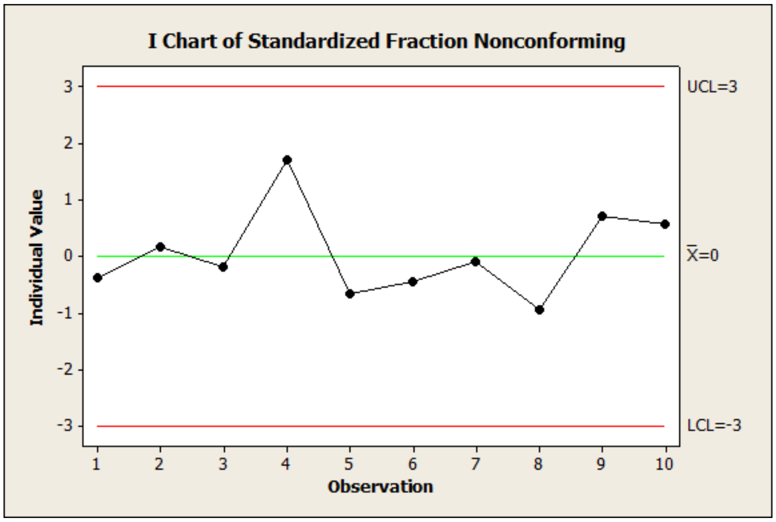
Construct a standardized control chart for the data in Exercise 7.11.

$$\bar{p} = 0.06$$

$$z_i = (\hat{p}_i - 0.06) \Big/ \sqrt{0.06(1-0.06)/n_i} = (\hat{p}_i - 0.06) \Big/ \sqrt{0.0564/n_i}$$

MTB > Stat > Control Charts > Variables Charts for Individuals > Individuals

Select I Chart Options, Parameters tab, to enter a Mean of 0 and Standard deviation of 1



The process is in control; results are the same as for the *p* chart.

7.48.

Surface defects have been counted on 26 rectangular steel plates, and the data are shown in Table 7E.12. Set up a control chart for nonconformities using these data. Does the process producing the plates appear to be in statistical control?

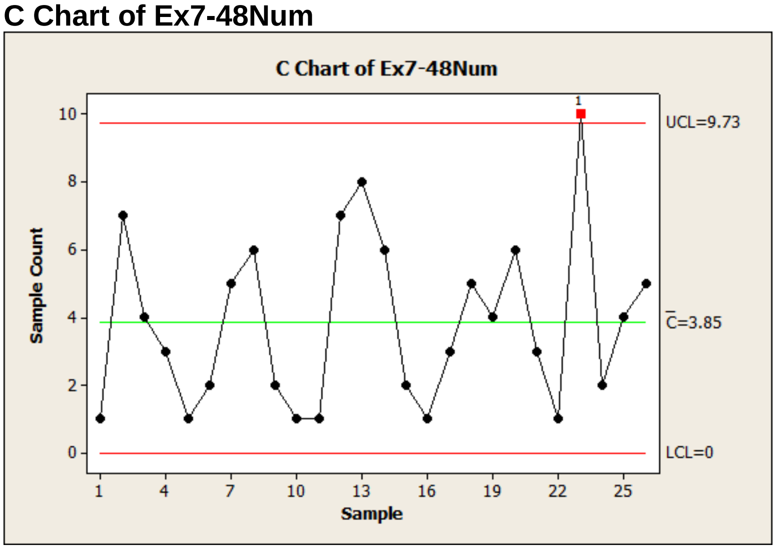
TABLE 7E.12
Data for Exercise 7.48

Plate Number	Number of Nonconformities	Plate Number	Number of Nonconformities
1	1	14	6
2	7	15	2
3	4	16	1
4	3	17	3
5	1	18	5
6	2	19	4
7	5	20	6
8	6	21	3
9	2	22	1
10	1	23	10
11	1	24	2
12	7	25	4
13	8	26	5

$$\text{CL} = \bar{c} = 3.85; \quad \text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 3.85 + 3\sqrt{3.85} = 9.74; \quad \text{LCL} = \bar{c} - 3\sqrt{\bar{c}} = 3.85 - 3\sqrt{3.85} = -2.04 \Rightarrow 0$$

MTB > Stat > Control Charts > Attributes Charts > C

C Chart of Ex7-48Num



Test Results for C Chart of Ex7-48Num

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  23
```

No. In addition to the point beyond the UCL, there appears to be a cyclical pattern in the plot points. The plate process does not seem to be in statistical control.

7.49.

A paper mill uses a control chart to monitor the imperfection nin finished rolls of paper. Production output is inspected for 20 days, and the resulting data are shown in Table 7E.13. Use these data to set up a control chart for nonconformities per roll of paper. Does the process appear to be in statistical control? What center line and control limits would you recommend for controlling current production?

TABLE 7E.13

Data on Imperfections in Rolls of Paper

Day	Number of Rolls Produced	Total Number of Imperfections	Day	Number of Rolls Produced	Total Number of Imperfections
1	18	12	11	18	18
2	18	14	12	18	14
3	24	20	13	18	9
4	22	18	14	20	10
5	22	15	15	20	14
6	22	12	16	20	13
7	20	11	17	24	16
8	20	15	18	24	18
9	20	12	19	22	20
10	20	10	20	21	17

$$\text{CL} = \bar{u} = 0.7007$$

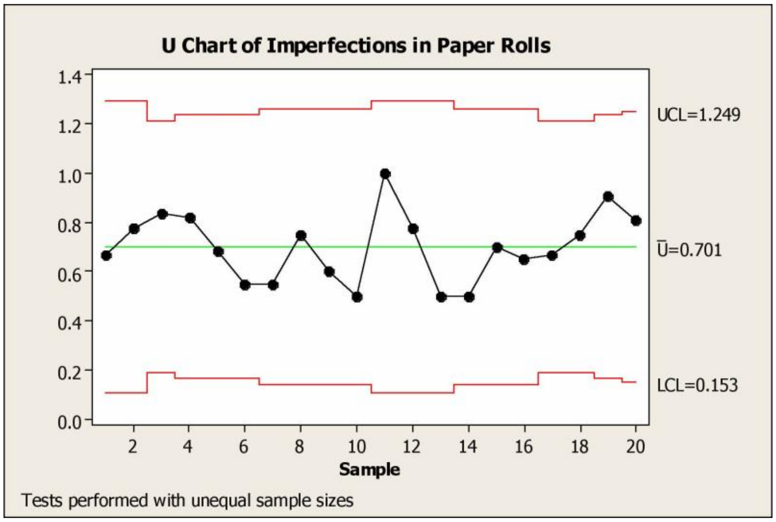
$$\text{UCL}_i = \bar{u} + 3\sqrt{\bar{u}/n_i} = 0.7007 + 3\sqrt{0.7007/n_i}$$

$$\text{LCL}_i = \bar{u} - 3\sqrt{\bar{u}/n_i} = 0.7007 - 3\sqrt{0.7007/n_i}$$

n_i	$[\text{LCL}_i, \text{UCL}_i]$
18	[0.1088, 1.2926]
20	[0.1392, 1.2622]
21	[0.1527, 1.2487]
22	[0.1653, 1.2361]
24	[0.1881, 1.2133]

7.49. continued

MTB > Stat > Control Charts > Attributes Charts > U



The process is in statistical control, with no patterns or out-of-control points. Use a control chart with CL = 0.701, UCL = 1.249, and LCL = 0.153 to control current production.

7.50.

Continuation of 7.49. Consider the paper-making process in Exercise 7.49. Set up a *u* chart based on an average sample size to control this process.

$$\text{CL} = \bar{u} = 0.7007; \quad \bar{n} = 20.55$$
$$\text{UCL} = \bar{u} + 3\sqrt{\bar{u}/\bar{n}} = 0.7007 + 3\sqrt{0.7007/20.55} = 1.2547$$
$$\text{LCL} = \bar{u} - 3\sqrt{\bar{u}/\bar{n}} = 0.7007 - 3\sqrt{0.7007/20.55} = 0.1467$$

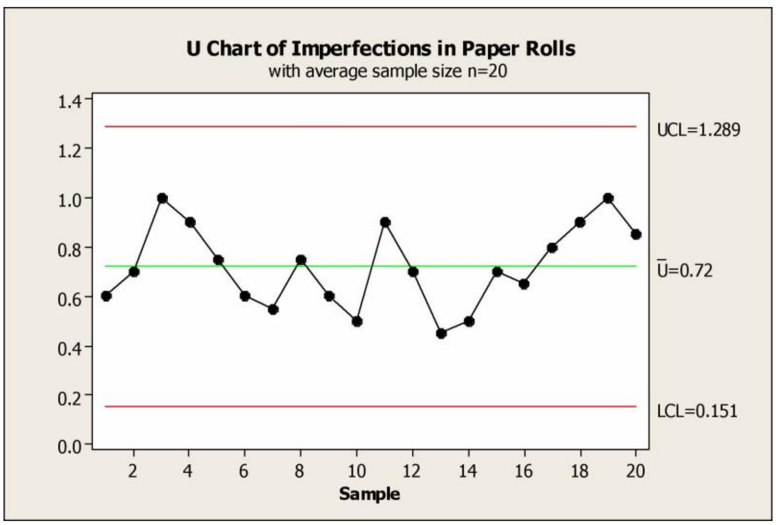
MTB > Stat > Basic Statistics > Display Descriptive Statistics

Descriptive Statistics: Ex7.37Rol

```
Variable    N    Mean
Ex7.37Rol  20  20.550
```

Average sample size is 20.55, however Minitab accepts only integer values for *n*. Use a sample size of *n* = 20, and carefully examine points near the control limits.

MTB > Stat > Control Charts > Attributes Charts > U



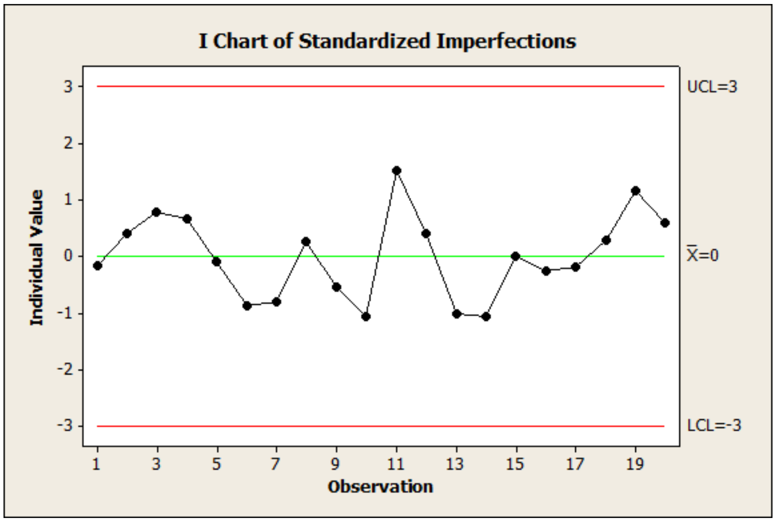
The process is in statistical control, with no patterns or out-of-control points.

7.51.

Continuation of Exercise7.49. Consider the paper-making process in Exercise 7.49. Set up a standardized *u* chart for this process.

$$z_i = (u_i - \bar{u}) \Big/ \sqrt{\bar{u}/n_i} = (u_i - 0.7007) \Big/ \sqrt{0.7007 / n_i}$$

MTB > Stat > Control Charts > Variables Charts for Individuals > Individuals
Select I Chart Options, Parameters tab, to enter a Mean of 0 and Standard deviation of 1



The process is in statistical control, with no patterns or out-of-control points.

7.52.

The number of nonconformities found on final inspection of a DVD player is shown in Table 7E.14. Can you conclude that the process is in statistical control? What center line and control limits would you recommend for controlling future production?

TABLE 7E.14
Data on Nonconformities in DVD Players

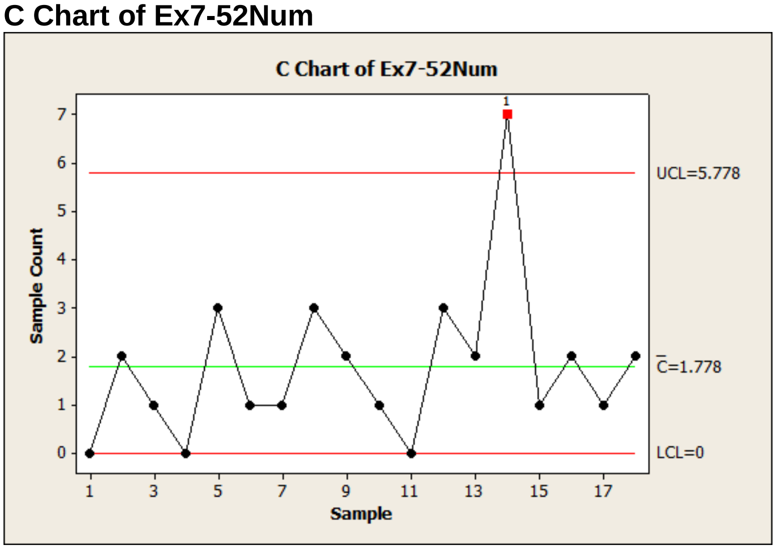
Player Number	Number of Nonconformities	Player Number	Number of Nonconformities
2412	0	2421	1
2413	2	2422	0
2414	1	2423	3
2415	0	2424	2
2416	3	2425	7
2417	1	2426	1
2418	1	2427	2
2419	3	2428	1
2420	2	2429	2

Utilize a *c* chart based on # of nonconformities per cassette deck.

$$\text{CL} = \bar{c} = 1.8; \quad \text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 1.8 + 3\sqrt{1.8} = 5.8; \quad \text{LCL} = \bar{c} - 3\sqrt{\bar{c}} = 1.8 - 3\sqrt{1.8} = -2.2 \Rightarrow 0$$

MTB > Stat > Control Charts > Attributes Charts > C

C Chart of Ex7-52Num



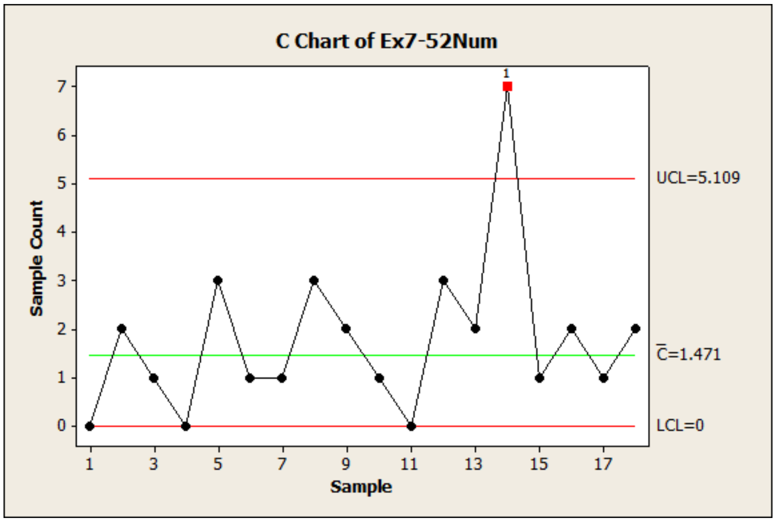
Test Results for C Chart of Ex7-52Num

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  14
```

Sample #14 signals out of control, exceeding the UCL. Assuming that an assignable cause is found, exclude this sample from calculation of control limits:

7.52. continued

C Chart of Ex7-52Num



Test Results for C Chart of Ex7-52Num

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  14
```

The remaining points are within the new control limits, without any obvious patterns or out-of-control points. For future production, use a *c* chart with center line = 1.5, UCL = 5.1, and LCL = 0.

7.53.

The data in Table 7E.15 represent the number of nonconformities per 1,000 meters in telephone cable. From analysis of these data, would you conclude that the process is in statistical control? What control procedure would you recommend for future production?

TABLE 7E.15

Telephone Cable Data for Exercise 7.53

Sample Number	Number of Nonconformities	Sample Number	Number of Nonconformities
1	1	12	6
2	1	13	9
3	3	14	11
4	7	15	15
5	8	16	8
6	10	17	3
7	5	18	6
8	13	19	7
9	0	20	4
10	19	21	9
11	24	22	20

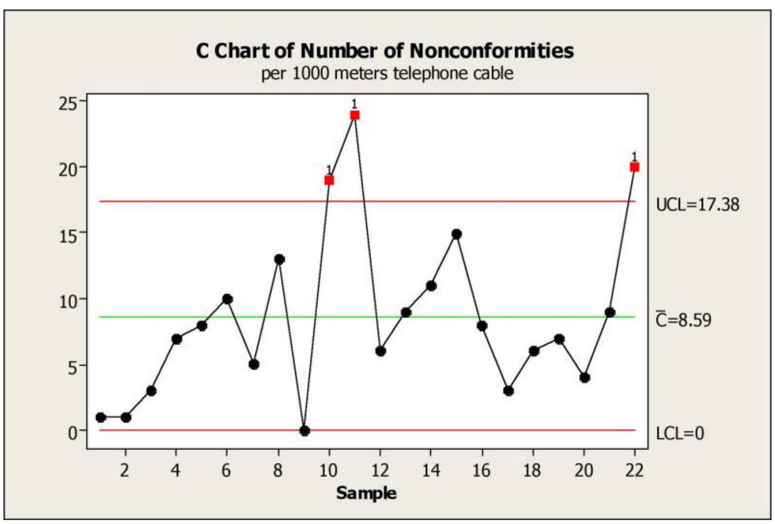
Utilize a *c* chart based on # of nonconformities per 1,000 meters.

$$\text{CL} = \bar{c} = 8.59$$

$$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 8.59 + 3\sqrt{8.59} = 17.384$$

$$\text{LCL} = \bar{c} - 3\sqrt{\bar{c}} = 8.59 - 3\sqrt{8.59} \Rightarrow 0$$

MTB > Stat > Control Charts > Attributes Charts > C



Test Results for C Chart of Ex7.41Num

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  10, 11, 22
```

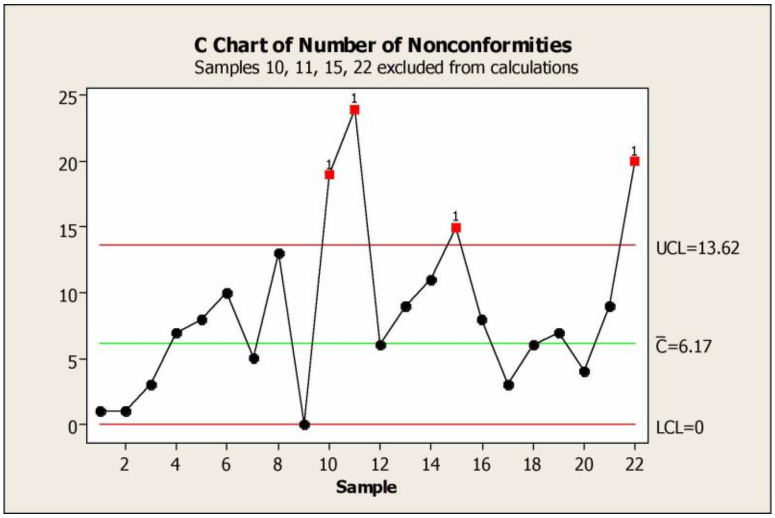
7.53. continued

The process is not in statistical control; three subgroups exceed the UCL. Exclude subgroups 10, 11 and 22, and re-calculate the control limits. Subgroup 15 will then be out of control and should also be excluded.

$$\text{CL} = \bar{c} = 6.17$$
$$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 6.17 + 3\sqrt{6.17} = 13.62$$
$$\text{LCL} \Rightarrow 0$$

MTB > Stat > Control Charts > Attributes Charts > C

To exclude subgroups, select C Chart Options, Estimate tab, Omit 10, 11, 15, 22 from calculations



Test Results for C Chart of Ex7.41Num

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  10, 11, 15, 22
```

For future production, recommend using a *c* chart with center line = 6.17, UCL = 13.62, and LCL = 0.

7.54.
Consider the data in Exercise 7.52. Suppose we wish to define a new inspection unit of five DVD players.

(a) What are the center line and control limits for a control chart for monitoring future production based on the total number of defects in the new inspection unit?

The new inspection unit is n = 5 DVD players. A c chart of the total number of nonconformities per inspection unit is appropriate. Use $\bar{c} = 1.5$ from the production control limits.

$$\text{CL} = n\bar{c} = 5(1.5) = 7.5; \quad \text{UCL} = n\bar{c} + 3\sqrt{n\bar{c}} = 7.5 + 3\sqrt{7.5} = 15.7; \quad \text{LCL} = n\bar{c} - 3\sqrt{n\bar{c}} = 7.5 - 3\sqrt{7.5} = -0.7 \Rightarrow 0$$

(b) What are the center line and control limits for a control chart for nonconformities per unit used to monitor future production.

The sample is n =1 new inspection units. A u chart of average nonconformities per inspection unit is appropriate.

$$\text{CL} = \bar{u} = \bar{c}_{\text{production}} \times \#\ \text{DVD players} = 1.5 \times 5 = 7.5$$
$$\text{UCL} = \bar{u} + 3\sqrt{\bar{u}/n} = 7.5 + 3\sqrt{7.5/1} = 15.7$$
$$\text{LCL} = \bar{u} - 3\sqrt{\bar{u}/n} = 7.5 - 3\sqrt{7.5/1} = -0.7 \Rightarrow 0$$

7.55.
Consider the data in Exercise 7.53. Suppose a new inspection unit is defined as 2,500 m of wire.

(a) What are the center line and control limits for a control chart for monitoring future production based on the total number of nonconformities in the new inspection unit?

The new inspection unit is n = 2500/1000 = 2.5 of the old unit. A c chart of the total number of nonconformities per inspection unit is appropriate.

$$\text{CL} = n\bar{c} = 2.5(6.17) = 15.43$$
$$\text{UCL} = n\bar{c} + 3\sqrt{n\bar{c}} = 15.43 + 3\sqrt{15.43} = 27.21$$
$$\text{LCL} = n\bar{c} - 3\sqrt{n\bar{c}} = 15.43 - 3\sqrt{15.43} = 3.65$$

The plot point, $\hat{c}$, is the total number of nonconformities found while inspecting a sample 2500m in length.

(b) What are the center line and control limits for a control chart for average nonconformities per unit used to monitor future production?

The sample is n =1 new inspection units. A u chart of average nonconformities per inspection unit is appropriate.

$$\text{CL} = \bar{u} = \frac{\text{total nonconformities}}{\text{total inspection units}} = \frac{111}{(18 \times 1000)/2500} = 15.42$$
$$\text{UCL} = \bar{u} + 3\sqrt{\bar{u}/n} = 15.42 + 3\sqrt{15.42/1} = 27.20$$
$$\text{LCL} = \bar{u} - 3\sqrt{\bar{u}/n} = 15.42 - 3\sqrt{15.42/1} = 3.64$$

The plot point, $\hat{u}$, is the average number of nonconformities found in 2500m, and since n = 1, this is the same as the total number of nonconformities.

7.56.

An electronics manufacturer wishes to control the number of nonconformities in a subassembly area producing mother boards. The inspection unit is defined as nine mother boards, and data from 16 samples (each of size 9) are shown in Table 7E.16.

TABLE 7E.16

Data for Exercise 7.56

Sample Number	Number of Nonconformities	Sample Number	Number of Nonconformities
1	1	9	2
2	3	10	1
3	2	11	0
4	1	12	2
5	0	13	1
6	2	14	1
7	1	15	2
8	5	16	3

(a) Set up a control chart for nonconformities per unit.

A *u* chart of average number of nonconformities per unit is appropriate, with *n* = 9 mother boards in each inspection.

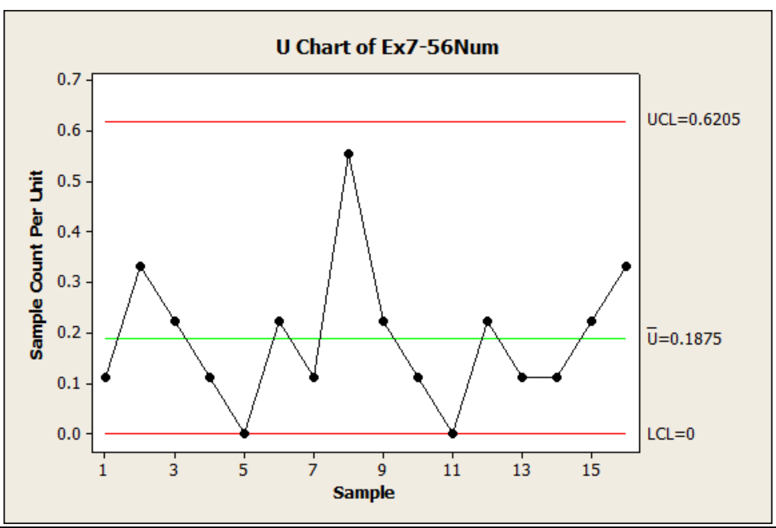
$$\text{CL} = \bar{u} = \sum u_i / m = \left(\sum x_i / n\right)/m = (27/9)/16 = 0.1875$$

$$\text{UCL} = \bar{u} + 3\sqrt{\bar{u}/n} = 0.1875 + 3\sqrt{0.1875/9} = 0.6205$$

$$\text{LCL} = \bar{u} - 3\sqrt{\bar{u}/n} = 0.1875 - 3\sqrt{0.1875/9} = -0.2455 \Rightarrow 0$$

MTB > Stat > Control Charts > Attributes Charts > U

U Chart of Ex7-56Num



7.56. continued

(b) Do these data come from a controlled process? If not, assume that assignable causes can be found for all out-of-control points and calculate the revised control chart parameters.

The process is in statistical control, with no obvious patterns or out-of-control points.

(c) Suppose the inspection unit is redefined as five mother boards. Design an appropriate control chart for monitoring future production.

The new sample is n = 5/9 = 0.56 inspection units. However, since this chart was established for ***average*** nonconformities per unit, the same control limits may be used for future production with the new sample size. (If this was a c chart for ***total*** nonconformities in the sample, the control limits would need revision.)

7.57.
Find the three-sigma control limits for

(a) a c chart with process average equal to four nonconformities.

$$\text{CL} = \bar{c} = 4$$
$$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 4 + 3\sqrt{4} = 10$$
$$\text{LCL} = \bar{c} - 3\sqrt{\bar{c}} = 4 - 3\sqrt{4} \Rightarrow 0$$

(b) a u chart with c = 4 and n = 4.

$$c = 4; \quad n = 4$$
$$\text{CL} = \bar{u} = c / n = 4 / 4 = 1$$
$$\text{UCL} = \bar{u} + 3\sqrt{\bar{u}/n} = 1 + 3\sqrt{1/4} = 2.5$$
$$\text{LCL} = \bar{u} - 3\sqrt{\bar{u}/n} = 1 - 3\sqrt{1/4} \Rightarrow 0$$

7.58.

Find 0.900 and 0.100 probability limits for a *c* chart when the process average is equal to 16 nonconformities.

Using the Poisson distribution:

$\bar{c} = 16$

$\Pr\{x \le 21 \mid c = 16\} = 0.9108; \text{UCL} = 21$

$\Pr\{x \le 10 \mid c = 16\} = 0.0774; \text{LCL} = 10$

MTB > Calc > Probability distributions > Poisson

Cumulative Distribution Function

```
Poisson with mean = 16
 x  P( X <= x )
10     0.077396
21     0.910773
```

7.59.

Find the three-sigma control limits for

(a) a *c* chart with process average equal to nine nonconformities.

$\text{CL} = \bar{c} = 9$

$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 9 + 3\sqrt{9} = 18$

$\text{LCL} = \bar{c} - 3\sqrt{\bar{c}} = 9 - 3\sqrt{9} = 0$

(b) a *u* chart with *c* = 16 and *n* = 4.

$c = 16; \quad n = 4$

$\text{CL} = \bar{u} = c / n = 16 / 4 = 4$

$\text{UCL} = \bar{u} + 3\sqrt{\bar{u}/n} = 4 + 3\sqrt{4/4} = 7$

$\text{LCL} = \bar{u} - 3\sqrt{\bar{u}/n} = 4 - 3\sqrt{4/4} = 1$

7.60.

Find 0.980 and 0.020 probability limits for a control chart for nonconformities per unit when u = 6.0 and n = 3.

u chart with u = 6.0 and n = 3. $c = u \times n$ = 18. Find limits such that Pr{$D \leq$ UCL} = 0.980 and Pr{D < LCL} = 0.020.

Using the Poisson distribution to find Pr{$D \leq$ x | c = 18}

MTB > Calc > Probability distributions > Poisson

Cumulative Distribution Function

```
Poisson with mean = 18
 x  P( X <= x )
 9     0.015381
10     0.030366
26     0.971766
27     0.982682
```

UCL = x/n = 27/3 = 9, and LCL = x/n = 9/3 = 3.

As a comparison, the normal distribution gives:

$$\text{UCL} = \bar{u} + z_{0.980}\sqrt{\bar{u}/n} = 6 + 2.054\sqrt{6/3} = 8.905$$

$$\text{LCL} = \bar{u} + z_{0.020}\sqrt{\bar{u}/n} = 6 - 2.054\sqrt{6/3} = 3.095$$

7.61.

Find 0.975 and 0.025 probability limits for a control chart for nonconformities when *c* = 7.6.

Using the cumulative Poisson distribution:

MTB > Calc > Probability distributions > Poisson

Cumulative Distribution Function

```
Poisson with mean = 7.6
 x  P( X <= x )
 2     0.018757
 3     0.055371
12     0.953566
13     0.976247
```

for the *c* chart, UCL = 13 and LCL = 2. As a comparison, the normal distribution gives

$$\text{UCL} = \bar{c} + z_{0.975}\sqrt{\bar{c}} = 7.6 + 1.96\sqrt{7.6} = 13.00$$

$$\text{LCL} = \bar{c} - z_{0.025}\sqrt{\bar{c}} = 7.6 - 1.96\sqrt{7.6} = 2.20$$

7.62.

A control chart for nonconformities per unit uses 0.95 and 0.05 probability limits. The center line is at *u* = 1.4. Determine the control limits if the sample size is *n* = 10.

Using the cumulative Poisson distribution with *c* = *u n* = 1.4(10) = 14, to find $\Pr\{D \leq x \mid c = 14\}$:

MTB > Calc > Probability distributions > Poisson

Cumulative Distribution Function

```
Poisson with mean = 14
 x  P( X <= x )
 7     0.031620
 8     0.062055
19     0.923495
20     0.952092
```

UCL = *x*/*n* = 20/10 = 2.00, and LCL = *x*/*n* = 7/10 = 0.70.

As a comparison, the normal distribution gives:

$$\text{UCL} = \bar{u} + z_{0.95}\sqrt{\bar{u}/n} = 1.4 + 1.645\sqrt{1.4/10} = 2.016$$

$$\text{LCL} = \bar{u} + z_{0.05}\sqrt{\bar{u}/n} = 1.4 - 1.645\sqrt{1.4/10} = 0.784$$

7.63.

The number of workmanship nonconformities observed in the final inspection of disk-drive assemblies has been tabulated as shown in Table 7E.17. Does the process appear to be in control?

TABLE 7E.17

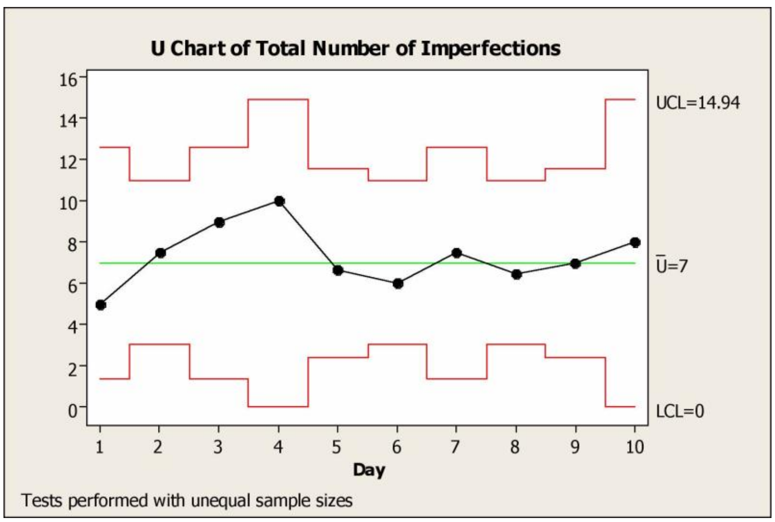
Data for Exercise 7.63

Day	Number of Assemblies Inspected	Total Number of Imperfections	Day	Number of Assemblies Inspected	Total Number of Imperfections
1	2	10	6	4	24
2	4	30	7	2	15
3	2	18	8	4	26
4	1	10	9	3	21
5	3	20	10	1	8

The appropriate chart is a *u* chart with control limits based on each sample size:

$$\bar{u} = 7; \quad \text{UCL}_i = 7 + 3\sqrt{7/n_i}; \quad \text{LCL}_i = 7 - 3\sqrt{7/n_i}$$

MTB > Stat > Control Charts > Attributes Charts > U



The process is in statistical control.

7.64. ☺

Most corporations use external accounting and auditing firms for performing audits on their financial records. In medium to large businesses there may be a very large number of accounts to audit, so auditors often use a technique called audit sampling, in which a random sample of accounts are selected for auditing and the results used to draw conclusions about the organization's accounting practices. Table 7E.18 presents the results of an audit sampling process, in which 25 accounts were randomly selected and the number of posting errors found. Set up a control chart for nonconformities for this process. Is this process in statistical control?

TABLE 7E.18
Audit Sampling Data for Exercise 7.64

Account	Number of Posting Errors	Account	Number of Posting Errors
1	0	14	0
2	2	15	2
3	1	16	1
4	4	17	4
5	0	18	6
6	1	19	1
7	3	20	1
8	2	21	3
9	0	22	4
10	1	23	1
11	0	24	0
12	0	25	1
13	2		

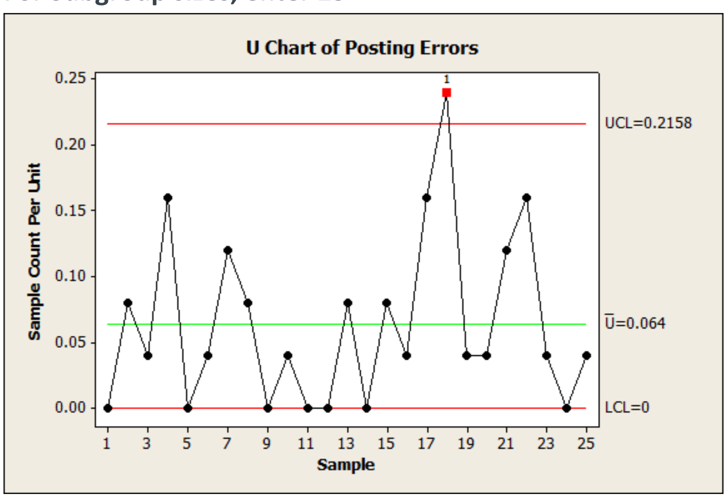
A *u* chart of average number of posting errors per account is appropriate, with *n* = 25 accounts in each audit.

$$\text{CL} = \bar{u} = \sum u_i / m = \left(\sum x_i / n\right)/m = (40/25)/25 = 0.064$$

$$\text{UCL} = \bar{u} + 3\sqrt{\bar{u}/n} = 0.064 + 3\sqrt{0.064/25} = 0.064 + 0.152 = 0.216$$

$$\text{LCL} = \bar{u} - 3\sqrt{\bar{u}/n} = 0.064 - 3\sqrt{0.064/25} = 0.064 - 0.152 \Rightarrow 0$$

MTB > Stat > Control Charts > Attributes Charts > U
For Subgroup sizes, enter 25



Test Results for U Chart of Ex7-64Err

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  18
```

The process is not in statistical control, with an out-of-control audit (# 18).

7.65. ☺

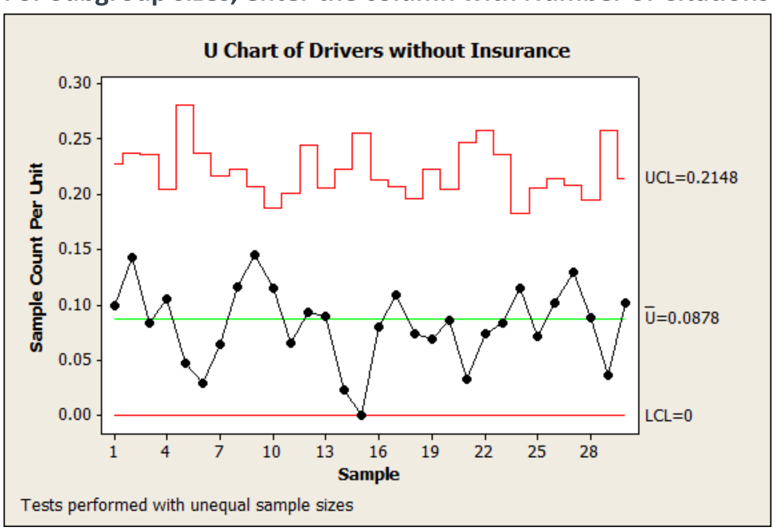
A metropolitan police agency is studying the incidence of drivers operating their vehicles without the minimum liability insurance required by law. The data are collected from drivers who have been stopped by an officer for a traffic law violation and a traffic summons issued. Data from three shifts over a ten-day period are shown in Table 7E.19.

TABLE 7E.19
Data for Exercise 7.65

Sample	Number of Citations	Number of Drivers Without Insurance	Sample	Number of Citations	Number of Drivers Without Insurance
1	40	4	16	50	4
2	35	5	17	55	6
3	36	3	18	67	5
4	57	6	19	43	3
5	21	1	20	58	5
6	35	1	21	31	1
7	47	3	22	27	2
8	43	5	23	36	3
9	55	8	24	87	10
10	78	9	25	56	4
11	61	4	26	49	5
12	32	3	27	54	7
13	56	5	28	68	6
14	43	1	29	27	1
15	28	0	30	49	5

(a) Set up a *u*-chart for these data. Plot the data from Table 7E.19 on the chart. Is the process in statistical control?

MTB > Stat > Control Charts > Attributes Charts > U
For Subgroup sizes, enter the column with Number of Citations



Test Results for U Chart of Ex7-64Err

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  18
```

The process is in control, with no out-of-control points over the ten-day period.

7.65. continued

(b) Are these data consistent with the hypothesis that about 10% of drivers operate without proper liability insurance coverage?

Use n = 30 shifts and the shift averages to perform a 1-sample t-test of the hypotheses:
H_0: $\mu = 10$ verus H_1: $\mu \neq 10$

MTB > Stat > Basic Statistics > 1-sample t

One-Sample T: Ex7-65Avg

```
Test of mu = 10 vs not = 10
Variable    N     Mean    StDev  SE Mean         95% CI                  T      P
Ex7-65Avg  30  0.08251  0.03509  0.00641  (0.06940, 0.09561)  -1548.00  0.000
```

The data are not consistent with the hypothesis that 10% of drivers do not have proper liability insurance coverage; the percentage is smaller.

7.66.

A control chart for nonconformities is to be constructed with c 2.0, LCL = 0, and UCL such that the probability of appoint plotting outside control limits when c = 2.0 is only 0.005.

(a) Find the UCL.

From the cumulative Poisson table, $\Pr\{x \leq 6 \mid c = 2.0\} = 0.995$. So set UCL = 6.0.

(b) What is the type I error probability of the process is assumed to be out of control only when two consecutive points fall outside the control limits?

Pr{two consecutive out-of-control points} = (0.005)(0.005) = 0.00003

7.67.

A textile mill wishes to establish a control procedure on flaws in towels it manufactures. Using an inspection unit of 50 units, past inspection data show that 100 previous units had 850 total flaws. What type of control chart is appropriate? Design the control chart such that it has two-sided probability control limits of α = 0.06, approximately. Give the center line and control limits.

A *c* chart with one inspection unit equal to 50 manufacturing units is appropriate. $\bar{c} = 850/100 = 8.5$.

MTB > Calc > Probability distributions > Poisson

Cumulative Distribution Function

```
Poisson with mean = 8.5
 x  P( X <= x )
 3     0.030109
13     0.948589
14     0.972575
```

LCL = 3 and UCL = 13. For comparison, the normal distribution gives

$$\text{UCL} = \bar{c} + z_{0.97}\sqrt{\bar{c}} = 8.5 + 1.88\sqrt{8.5} = 13.98$$

$$\text{LCL} = \bar{c} + z_{0.03}\sqrt{\bar{c}} = 8.5 - 1.88\sqrt{8.5} = 3.02$$

7.68.

The manufacturer wishes to set up a control chart at the final inspection station for a vending machine. Defects in workmanship and visual quality features are checked in this inspection. For the past 22 working days, 198 machines were inspected and a total of 1089 nonconformities reported.

(a) What type of control chart would you recommend here, and how would you use it?

Plot the number of nonconformities per water heater on a *c* chart.

$$\text{CL} = \bar{c} = \sum D/m = 1089/198 = 5.5$$

$$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 5.5 + 3\sqrt{5.5} = 12.5$$

$$\text{LCL} = \bar{c} - 3\sqrt{\bar{c}} = 5.5 - 3\sqrt{5.5} = -1.5 \Rightarrow 0$$

Plot the results after inspection of each water heater, approximately 9/day.

(b) Using two machines as the inspection unit, calculate the center line and control limits that are consistent with the past 22 days of inspection data.

Let new inspection unit *n* = 2 water heaters

$$\text{CL} = n\bar{c} = 2(5.5) = 11$$

$$\text{UCL} = n\bar{c} + 3\sqrt{n\bar{c}} = 11 + 3\sqrt{11} = 20.9$$

$$\text{LCL} = n\bar{c} - 3\sqrt{n\bar{c}} = 11 - 3\sqrt{11} = 1.1$$

7.68. continued

(c) What is the probability of type I error for the control chart in part (b)?

$$\begin{aligned}\Pr\{\text{type I error}\} &= \Pr\{D < \text{LCL} \mid c\} + \Pr\{D > \text{UCL} \mid c\} \\ &= \Pr\{D < 1.1 \mid 11\} + \left[1 - \Pr\{D \le 20.9 \mid 11\}\right] \\ &= \Pr\{D < 1.1 \mid 11\} + \left[1 - \Pr\{D \le 20.9 \mid 11\}\right] \\ &= \text{POI}(1,11) + \left[1 - \text{POI}(20,11)\right] \\ &= 0.0002 + \left[1 - 0.9953\right] \\ &= 0.0049\end{aligned}$$

7.69.

Assembled portable television sets are subjected to a final inspection for surface defects. A total procedure is established based on the requirement that if the average number of nonconformities per units is 4.0, theprobability of concluding that the process is in control will be 0.99. There is to be no lower control limit. What is the appropriate type of control chart and what is the required upper limit?

$\bar{u} = 4.0$ average number of nonconformities/unit. Desire α = 0.99. Use the cumulative Poisson distribution to determine the UCL:

MTB > Calc > Probability distributions > Poisson

Cumulative Distribution Function

```
Poisson with mean = 4
 x  P( X <= x )
 0     0.018316
 1     0.091578
 2     0.238103
 3     0.433470
 4     0.628837
 5     0.785130
 6     0.889326
 7     0.948866
 8     0.978637
 9     0.991868
10     0.997160
11     0.999085
```

An UCL = 9 will give a probability of 0.99 of concluding the process is in control, when in fact it is.

7.70.

A control chart is to be established on a process producing refrigerators. The inspection unit is one refrigerator, and a common chart for nonconformities is to be used. As preliminary data, 16 nonconformities were counted in inspecting 30 refrigerators.

Use a c chart for nonconformities with an inspection unit n = 1 refrigerator.

$\sum D_i = 16$ in 30 refrigerators; $\bar{c} = 16/30 = 0.533$

(a) What are the three-sigma control limits?

3-sigma limits are $\bar{c} \pm 3\sqrt{\bar{c}} = 0.533 \pm 3\sqrt{0.533} = [0, 2.723]$

(b) What is the α-risk for this control chart?

$$\begin{aligned}
\alpha &= \Pr\{D < \text{LCL} \mid c\} + \Pr\{D > \text{UCL} \mid c\} \\
&= \Pr\{D < 0 \mid 0.533\} + \left[1 - \Pr\{D \le 2.72 \mid 0.533\}\right] \\
&= 0 + \left[1 - \text{POI}(2, 0.533)\right] \\
&= 1 - 0.983 \\
&= 0.017
\end{aligned}$$

where POI(·) is the cumulative Poisson distribution.

(c) What is the β-risk if the average number of defects is actually 2 (i.e., if c = 2.0)?

$$\begin{aligned}
\beta &= \Pr\{\text{not detecting shift}\} \\
&= \Pr\{D < \text{UCL} \mid c\} - \Pr\{D \le \text{LCL} \mid c\} \\
&= \Pr\{D < 2.72 \mid 2.0\} - \Pr\{D \le 0 \mid 2.0\} \\
&= \text{POI}(2,2) - \text{POI}(0,2) \\
&= 0.6767 - 0.1353 \\
&= 0.5414
\end{aligned}$$

where POI(·) is the cumulative Poisson distribution.

(d) Find the average run length if the average number o defects is actually 2.

$$\text{ARL}_1 = \frac{1}{1-\beta} = \frac{1}{1-0.541} = 2.18 \approx 2$$

7.71.

Consider the situation described in Exercise 7.70. ($\bar{c} = 0.533$)

(a) Find two-sigma control limits and compare these with the control limits found in art (a) of exercise 7.70.

$$\bar{c} \pm 2\sqrt{\bar{c}} = 0.533 + 2\sqrt{0.533} = [0, 1.993]$$

(b) Find the α-risk for the control chart with two-sigma control limits and compare with the results of part (b) of Exercise 7.70.

$$\begin{aligned}\alpha &= \Pr\{D < \text{LCL} \mid \bar{c}\} + \Pr\{D > \text{UCL} \mid \bar{c}\}\\ &= \Pr\{D < 0 \mid 0.533\} + \left[1 - \Pr\{D \le 1.993 \mid 0.533\}\right]\\ &= 0 + \left[1 - \text{POI}(1, 0.533)\right]\\ &= 1 - 0.8996\\ &= 0.1004\end{aligned}$$

where POI(·) is the cumulative Poisson distribution.

(c) Find the β-risk for *c* – 2.0 for the chart with two-sigma control limits and compare with the results of part (c) of Exercise 7.70.

$$\begin{aligned}\beta &= \Pr\{D < \text{UCL} \mid c\} - \Pr\{D \le \text{LCL} \mid c\}\\ &= \Pr\{D < 1.993 \mid 2\} - \Pr\{D \le 0 \mid 2\}\\ &= \text{POI}(1, 2) - \text{POI}(0, 2)\\ &= 0.406 - 0.135\\ &= 0.271\end{aligned}$$

where POI(·) is the cumulative Poisson distribution.

(d) Find the ARL if *c* = 2.0 and compare with the ARL found in part (d) of Exercise 7.70.

$$\text{ARL}_1 = \frac{1}{1-\beta} = \frac{1}{1-0.271} = 1.372 \approx 2$$

7.72.

A control chart for nonconformities is to be established in conjunction with final inspection of a radio. The inspection unit is to be a group of ten radios. The average number of nonconformities per radio had, in the past, been 0.5. Find three-sigma control limits for a *c* chart based on this size inspection unit.

1 inspection unit = 10 radios, $\bar{u} = 0.5$ average nonconformities/radio

$$\text{CL} = \bar{c} = \bar{u} \times n = 0.5(10) = 5$$
$$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 5 + 3\sqrt{5} = 11.708$$
$$\text{LCL} \Rightarrow 0$$

7.73.

A control chart for nonconformities is maintained on a process producing desk calculators. The inspection unit is defined as two calculators. The average number of nonconformities per machine when the process is in control is estimated to be two.

$\bar{u} = \text{average \# nonconformities/calculator} = 2$

(a) Find the appropriate three-sigma control limits for this size inspection unit.

c chart with $\bar{c} = \bar{u} \times n = 2(2) = 4$ nonconformities/inspection unit

$$\text{CL} = \bar{c} = 4$$
$$\text{UCL} = \bar{c} + k\sqrt{\bar{c}} = 4 + 3\sqrt{4} = 10$$
$$\text{LCL} = \bar{c} - k\sqrt{\bar{c}} = 4 - 3\sqrt{4} \Rightarrow 0$$

(b) What is the probability of type I error for this control chart?

Type I error =

$$\begin{aligned}\alpha &= \Pr\{D < \text{LCL} \mid \bar{c}\} + \Pr\{D > \text{UCL} \mid \bar{c}\} \\ &= \Pr\{D < 0 \mid 4\} + \left[1 - \Pr\{D \le 10 \mid 4\}\right] \\ &= 0 + \left[1 - \text{POI}(10,4)\right] \\ &= 1 - 0.997 \\ &= 0.003\end{aligned}$$

where POI(·) is the cumulative Poisson distribution.

7.74.

A production line assembles food mixers. The average number of nonconformities per mixer is estimated to be 0.65. The quality engineer wishes to establish a *c* chart for this operation, using an inspection unit of eight mixers. Find the three-sigma limits for this chart.

1 inspection unit = 8 mixers, $\bar{u} = 0.65$ nonconformities/clock

$$\text{CL} = \bar{c} = \bar{u} \times n = 0.65(8) = 5.2$$
$$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 5.2 + 3\sqrt{5.2} = 12.0$$
$$\text{LCL} = \bar{c} - 3\sqrt{\bar{c}} = 5.2 - 3\sqrt{5.2} = -1.6 \Rightarrow 0$$

7.75.

Suppose that we wish to design a control chart for nonconformities per unit with *L*-sigma limits. Find the minimum sample size that would result in a positive lower control limit for this chart.

c: nonconformities per unit; *L*: sigma control limits

$$n\bar{c} - L\sqrt{n\bar{c}} > 0$$
$$n\bar{c} > L\sqrt{n\bar{c}}$$
$$n > L^2 / \bar{c}$$

7.76.

Kittlitz (1999) presents data on homicides in Waco, Texas, for the years 1980-1989 (data taken from the *Waco Tribune-Herald*, December 29, 1989). There were 29 homicides in 1989. Table 7E.20 gives the dates of the 1989 homicides and the number of days between each homicide.

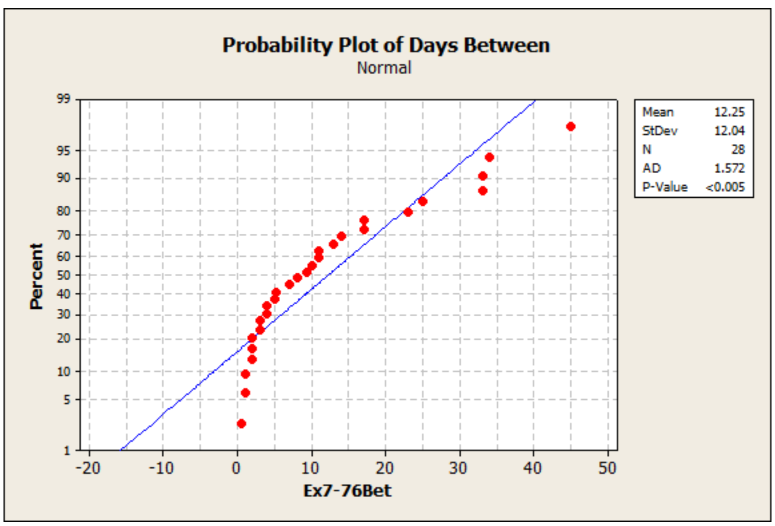
TABLE 7E.20

Homicide Data from Waco, Texas, for Exercise 7.76

Month	Date	Days Between	Month	Date	Days Between
Jan.	20		July	8	2
Feb.	23	34	July	9	1
Feb.	25	2	July	26	17
March	5	8	Sep.	9	45
March	10	5	Sep.	22	13
April	4	25	Sep.	24	2
May	7	33	Oct.	1	7
May	24	17	Oct.	4	3
May	28	4	Oct.	8	4
June	7	10	Oct.	19	11
June	16*	9.25	Nov.	2	14
June	16*	0.50	Nov.	25	23
June	22*	5.25	Dec.	28	33
June	25	3	Dec.	29	1
July	6	11			

(a) Plot the days-between-homicides data on a normal probability plot. Does the assumption of a normal distribution seem reasonable for these data?

MTB > Stat > Basic Statistics > Normality test

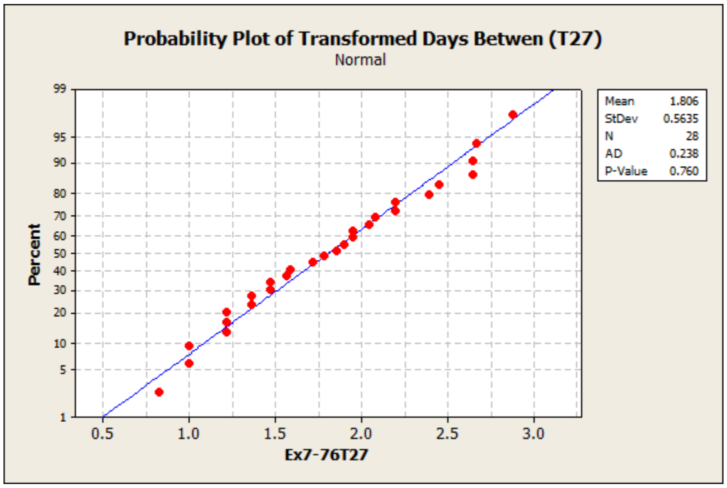


There is a huge curve in the plot points, indicating that the normal distribution assumption is not reasonable.

7.76. continued

(b) Transform the data using the 0.2777 root of the data. Plot the transformed data on a normal probability plot. Does this plot indicate that the transformation has been successful in making the new data more closely resemble data from a normal distribution?

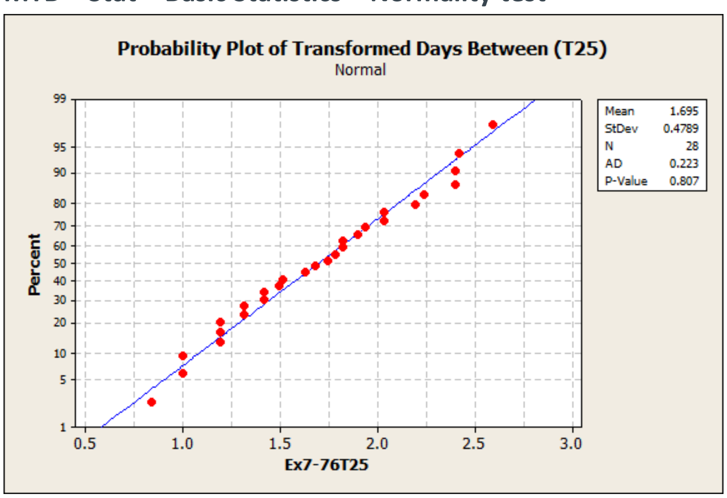
MTB > Stat > Basic Statistics > Normality test



The 0.2777[th] root transformation makes the data more closely resemble a sample from a normal distribution.

(c) Transform the data using the fourth root (0.25) of the data. Plot the transformed data on a normal probability plot. Does this plot indicate that the transformation has been successful in making the new data more closely resemble data from a normal distribution? Is the plot very different from the one in part (b)?

MTB > Stat > Basic Statistics > Normality test

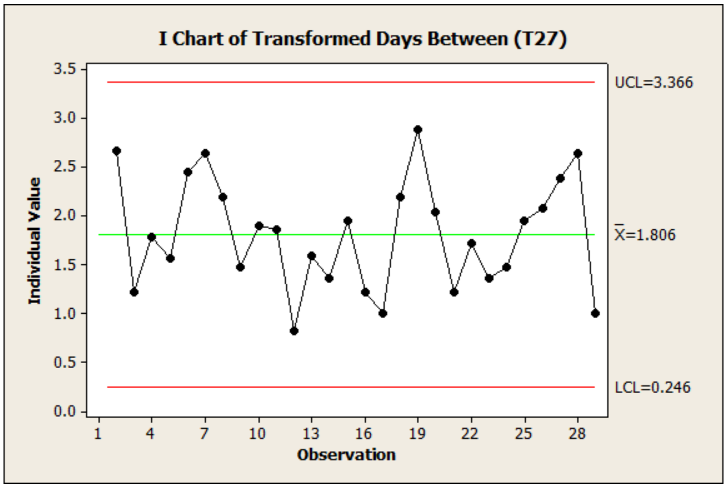


The 0.25[th] root transformation makes the data more closely resemble a sample from a normal distribution. It is not very different from the transformed data in (b).

7.76. continued

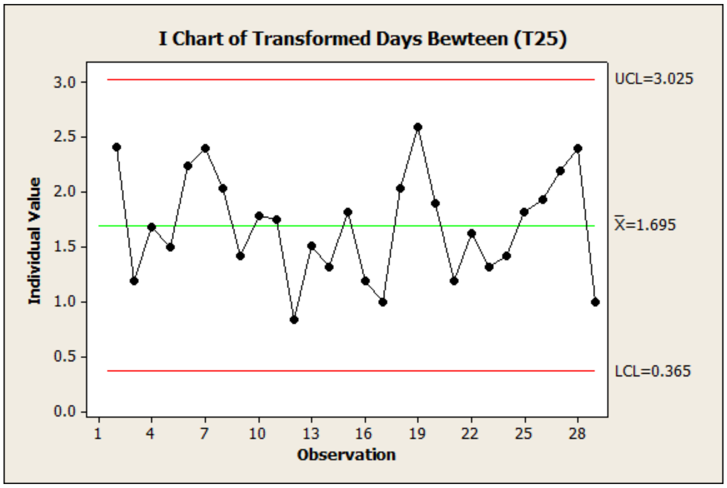
(d) Construct an individuals control chart using the transformed data from part (b).

MTB > Stat > Control Charts > Variables Charts for Individuals > Individuals



(e) Construct an individuals control chart using the transformed data from part (c). How similar is it to the one you constructed in part (d)?

MTB > Stat > Control Charts > Variables Charts for Individuals > Individuals



Both Individuals charts are similar, with an identical pattern of points relative to the UCL, mean and LCL. There is no difference in interpretation.

(f) Is the process stable? Provide a practical interpretation of the control chart.

The "process" is stable, meaning that the days-between-homicides is approximately constant. If a change is made, say in population, law, policy, workforce, etc., which affects the rate at which homicides occur, the mean time between may get longer (or shorter) with plot points above the upper (or below the lower) control limit.

7.77.
Suggest at least two nonmanufacturing scenarios in which attributes control charts could be useful for process monitoring.

There are endless possibilities for collection of attributes data from non-manufacturing processes. Consider a product distribution center (or any warehouse) with processes for filling and shipping orders. One could track the number of orders filled incorrectly (wrong parts, too few/many parts, wrong part labeling,), packaged incorrectly (wrong material, wrong package labeling), invoiced incorrectly, etc. Or consider an accounting firm—errors in statements, errors in tax preparation, etc. (hopefully caught internally with a verification step).

7.78.
What practical difficulties could be encountered in monitoring time-between-events data?

If time-between-events data (say failure time) is being sought for internally generated data, it can usually be obtained reliably and consistently. However, if you're looking for data on time-between-events that must be obtained from external sources (for example, time-to-field failures), it may be hard to determine with sufficient accuracy—both the "start" and the "end". Also, the conditions of use and the definition of "failure" may not be consistently applied.

There are ways to address these difficulties. Collection of "start" time data may be facilitated by serializing or date coding product.

7.79.

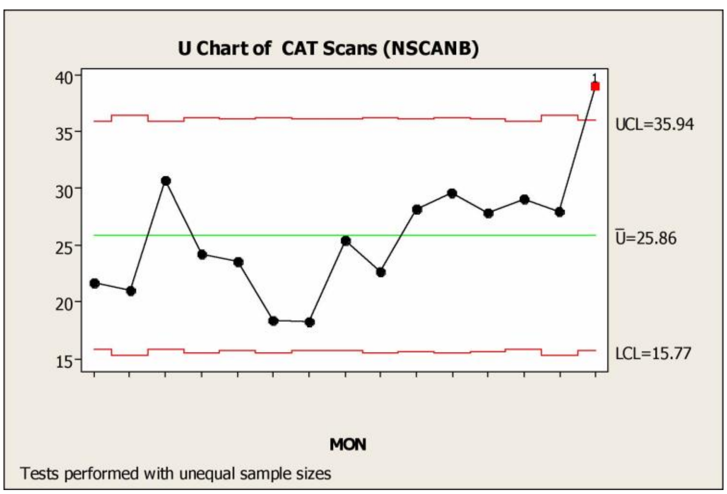
A paper by R.N. Rodriguez ("Health Care Applications of Statistical Process Control: Examples Using the SAS® System," *SAS Users Group International: Proceedings of the 21st Annual Conference*, 1996) illustrated several informative applications of control charts to the health care environment. One of these showed how a control chart was employed to analyze the rate of CAT scans performed each month at a clinic. The data used in this example are shown in Table 7E.21. NSCANB is the number of CAT scans performed each month and MMSB is the number of members enrolled in the health care plan each month, in units of member months. DAYS is the number of days in each month. The variable NYRSB converts MMSB to units of thousand members per year, and is computed as follows: NYRSB = MMSB (Days/30)/12000. NYRSB represents the "area of opportunity." Construct an appropriate control chart to monitor the rate at which CAT scans are performed at this clinic.

TABLE 7E.21
Data for Exercise 7.79

Month	NSCANB	MMSB	Days	NYRSB
Jan. 94	50	26,838	31	2.31105
Feb. 94	44	26,903	28	2.09246
March 94	71	26,895	31	2.31596
Apr. 94	53	26,289	30	2.19075
May 94	53	26,149	31	2.25172
Jun. 94	40	26,185	30	2.18208
July 94	41	26,142	31	2.25112
Aug. 94	57	26,092	31	2.24681
Sept. 94	49	25,958	30	2.16317
Oct. 94	63	25,957	31	2.23519
Nov. 94	64	25,920	30	2.16000
Dec. 94	62	25,907	31	2.23088
Jan. 95	67	26,754	31	2.30382
Feb. 95	58	26,696	28	2.07636
March 95	89	26,565	31	2.28754

The variable NYRSB can be thought of as an "inspection unit", representing an identical "area of opportunity" for each "sample". The "process characteristic" to be controlled is the rate of CAT scans. A *u* chart which monitors the average number of CAT scans per NYRSB is appropriate.

MTB > Stat > Control Charts > Attributes Charts > U



Test Results for U Chart of Ex7.65NSCANB

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  15
```

The rate of monthly CAT scans is out of control, exceeding the upper control limit in March'95.

7.80.

A paper by R.N. Rodriguez ("Health Care Applications of Statistical Process Control: Examples Using the SAS® System," *SAS Users Group International: Proceedings of the 21st Annual Conference*, 1996) illustrated several informative applications of control charts to the health care environment. One of these showed how a control chart was employed to analyze the number of office visits by health care plan members. The data for clinic E are shown in Table 7E.22. The variable NVISITE is the number of visits to clinic E each month, and MMSE is the number of members enrolled in the health care plan each month, in units of member months. DAYS is the number of days in each month. The variable NYRSE converts MMSE to units of thousand members per year, and is computed as follows: NYRSE = MMSE(Days/30)/12000. NYRSE represents the "area of opportunity." The variable PHASE separates the data into two time periods.

TABLE 7E.22
Data for Exercise 7.80

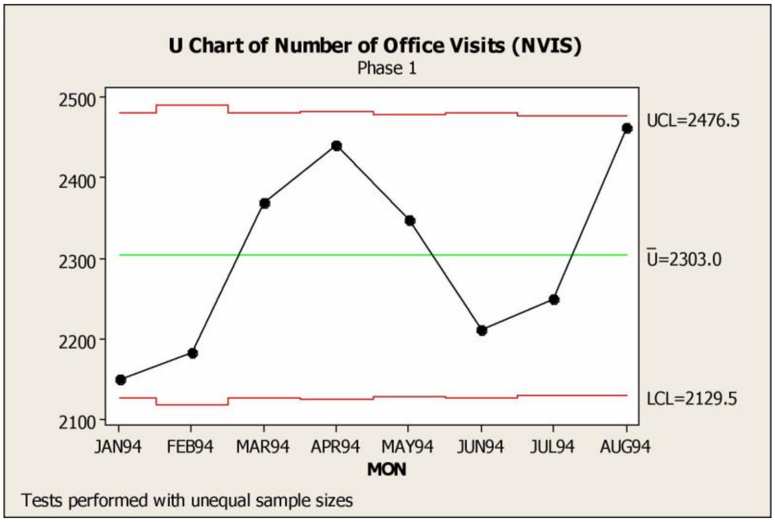
Month	Phase	NVISITE	NYRSE	Days	MMSE
Jan. 94	1	1,421	0.66099	31	7,676
Feb. 94	1	1,303	0.59718	28	7,678
Mar. 94	1	1,569	0.66219	31	7,690
Apr. 94	1	1,576	0.64608	30	7,753
May 94	1	1,567	0.66779	31	7,755
Jun. 94	1	1,450	0.65575	30	7,869
July 94	1	1,532	0.68105	31	7,909
Aug. 94	1	1,694	0.68820	31	7,992
Sep. 94	2	1,721	0.66717	30	8,006
Oct. 94	2	1,762	0.69612	31	8,084
Nov. 94	2	1,853	0.68233	30	8,188
Dec. 94	2	1,770	0.70809	31	8,223
Jan. 95	2	2,024	0.78215	31	9,083
Feb. 95	2	1,975	0.70684	28	9,088
Mar. 95	2	2,097	0.78947	31	9,168

The variable NYRSE can be thought of as an "inspection unit", representing an identical "area of opportunity" for each "sample". The "process characteristic" to be controlled is the rate of office visits. A *u* chart which monitors the average number of office visits per NYRSB is appropriate.

7.80. continued

(a) Use the data from Phase 1 to construct a control chart for monitoring the rate of office visits performed at clinic E. Does this chart exhibit control?

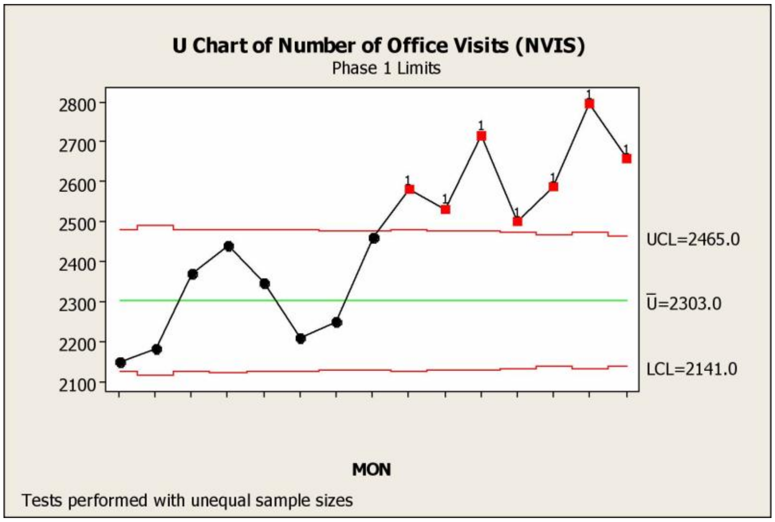
MTB > Stat > Control Charts > Attributes Charts > U



The chart is in statistical control

(b) Plot the data from Phase 2 on the chart constructed in part (a). Is there a difference in the two phases?

MTB > Stat > Control Charts > Attributes Charts > U



Test Results for U Chart of Ex7.66NVIS

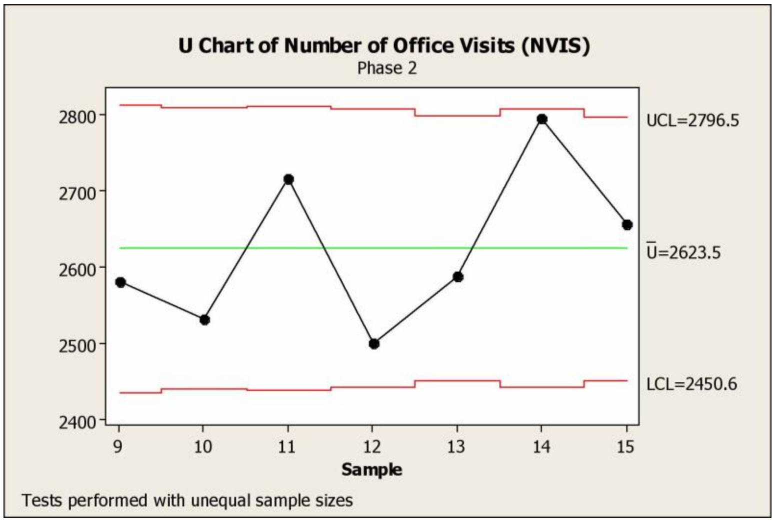
```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  9, 10, 11, 12, 13, 14, 15
```

The phase 2 data appears to have shifted up from phase 1. The 2nd phase is not in statistical control relative to the 1st phase.

7.80. continued

(c) Consider only the Phase 2 data. Do these data exhibit control?

MTB > Stat > Control Charts > Attributes Charts > U



The Phase 2 data, separated from the Phase 1 data, are in statistical control.

7.81.

The data in Table 7E.23 are the number of information errors found in customer records in a marketing company database. Five records were sampled each day.

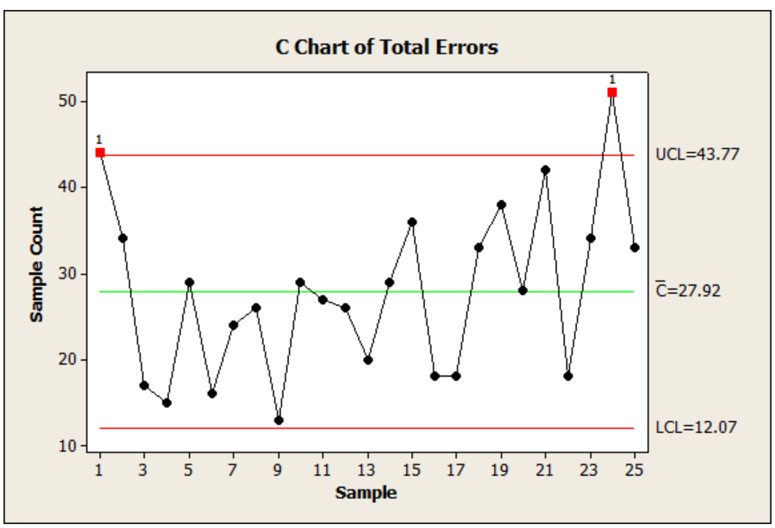
TABLE 7E.23

Customer Error Data for Exercise 7.81

Day	Record 1	Record 2	Record 3	Record 4	Record 5
1	8	7	1	11	17
2	11	1	11	2	9
3	1	1	8	2	5
4	3	2	5	1	4
5	3	2	13	6	5
6	6	3	3	3	1
7	8	8	2	1	5
8	4	10	2	6	4
9	1	6	1	3	2
10	15	1	3	2	8
11	1	7	13	5	1
12	6	7	9	3	1
13	7	6	3	3	1
14	2	9	3	8	7
15	6	14	7	1	8
16	2	9	4	2	1
17	11	1	1	3	2
18	5	5	19	1	3
19	6	15	5	6	6
20	2	7	9	2	8
21	7	5	6	14	10
22	4	3	8	1	2
23	4	1	4	20	5
24	15	2	7	10	17
25	2	15	3	11	2

(a) Set up a *c* chart for the total number of errors. Is the process in control?

MTB > Stat > Control Charts > Attributes Charts > C



Test Results for C Chart of Ex7-81Total

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  1, 24
```

The process has two points out of control, day 1 and day 24.

7.81. continued

(b) Set up a *t* chart for the total number of errors, assuming a geometric distribution with *a* = 1. Is the process in control?

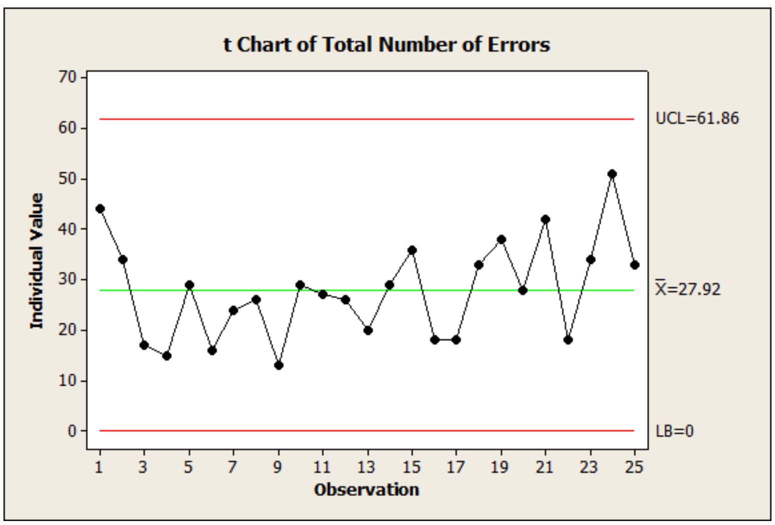
The limits for the t chart are given as

$$\text{UCL} = \bar{t} + 3\sqrt{n\left(\frac{\bar{t}}{n} - a\right)\left(\frac{\bar{t}}{n} - a + 1\right)} = 27.92 + 3\sqrt{5\left(\frac{27.92}{5} - 1\right)\left(\frac{27.92}{5} - 1 + 1\right)} = 27.92 + 3(11.3131) = 61.86$$

$$\text{CL} = \bar{t} = 27.92$$

$$\text{LCL} = \bar{t} - 3\sqrt{n\left(\frac{\bar{t}}{n} - a\right)\left(\frac{\bar{t}}{n} - a + 1\right)} = 27.92 - 3\sqrt{5\left(\frac{27.92}{5} - 1\right)\left(\frac{27.92}{5} - 1 + 1\right)} = 27.92 - 3(11.3131) - 6.02 \Rightarrow 0$$

MTB > Stat > Control Charts > Variables Charts for Individuals > Individuals
Under I Chart Options, on the Parameters tab, enter Mean = 27.92 and Standard deviation = 11.3131
Under I Chart Options, on the S limits tab, enter a lower bound for LCL of 0



Assuming a geometric distribution with *a* = 1, the process is in control.

(c) Discuss the findings from parts (a) and (b). Is the Poisson distribution a good model for the customer error data? Is there evidence of this in the data?

The geometric distribution seems to be a better fit to the error data. Since there are no records with 0 errors, the Poisson distribution does not seem to be a good fit.

7.82.

Kaminsky et al. (1992) present data on the number of orders per truck at a distribution center. Some of these data are sown in Table 7E.24.

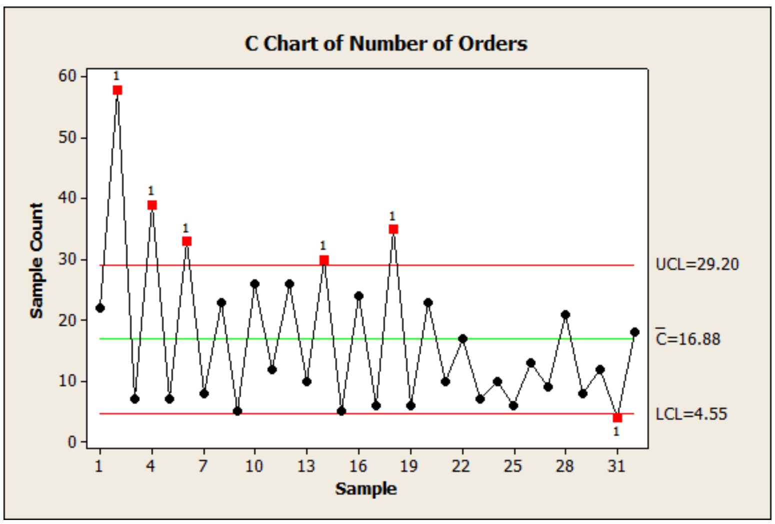
TABLE 7E.24

Number of Orders per Truck for Exercise 7.82

Truck	No. of Orders	Truck	No. of Orders	Truck	No. of Orders	Truck	No. of Orders
1	22	9	5	17	6	25	6
2	58	10	26	18	35	26	13
3	7	11	12	19	6	27	9
4	39	12	26	20	23	28	21
5	7	13	10	21	10	29	8
6	33	14	30	22	17	30	12
7	8	15	5	23	7	31	4
8	23	16	24	24	10	32	18

(a) Set up a *c* chart for the number of orders per truck. Is the process in control?

MTB > Stat > Control Charts > Attributes Charts > C



Test Results for C Chart of Ex7-82Ord

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  2, 4, 6, 14, 18, 31
```

The process is not in statistical control. It has 6 points that are outside of the control limits.

7.82. continued

(b) Set up a *t* chart for the number of orders per truck, assuming a geometric distribution with *a* = 1. Is the process in control?

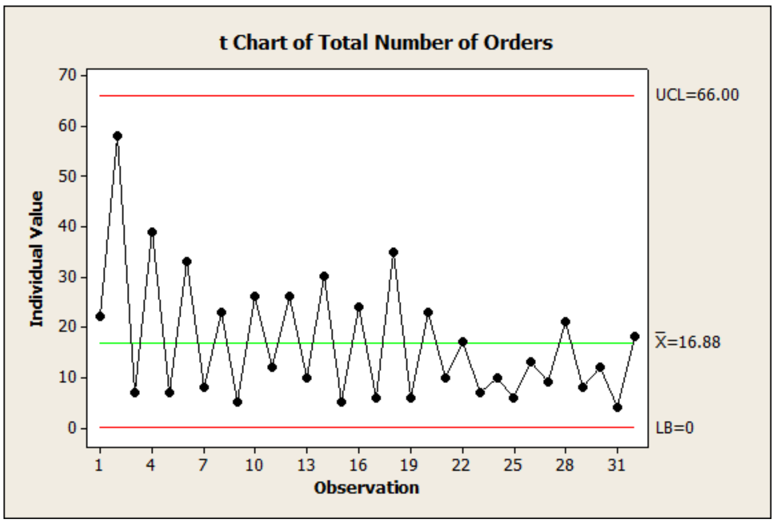
The limits for the t chart are given as

$$\text{UCL} = \bar{t} + 3\sqrt{n\left(\frac{\bar{t}}{n} - a\right)\left(\frac{\bar{t}}{n} - a + 1\right)} = 16.88 + 3\sqrt{1\left(\frac{16.88}{1} - 1\right)\left(\frac{16.88}{1} - 1 + 1\right)} = 16.88 + 3(16.3724) = 66.00$$

$$\text{CL} = \bar{t} = 16.88$$

$$\text{LCL} = \bar{t} - 3\sqrt{n\left(\frac{\bar{t}}{n} - a\right)\left(\frac{\bar{t}}{n} - a + 1\right)} = 16.88 - 3\sqrt{1\left(\frac{16.88}{1} - 1\right)\left(\frac{16.88}{1} - 1 + 1\right)} = 16.88 - 3(16.3724) - 32.24 \Rightarrow 0$$

MTB > Stat > Control Charts > Variables Charts for Individuals > Individuals
Under I Chart Options, on the Parameters tab, enter Mean = 16.88 and Standard deviation = 16.3724
Under I Chart Options, on the S limits tab, enter a lower bound for LCL of 0



Assuming a geometric distribution with *a* = 1, the process is in control.

(c) Discuss the findings from parts (a) and (b). Is the Poisson distribution a good model for these data? Is there evidence of this in the data?

Since the data does not have any 0 values, the Poisson is not a good fit for the data. The geometric distribution with a=1 appears to be a better fit.